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MENTAL FATIGUE

DURING CONTINUOUS EXERCISE OF
A SINGLE FUNCTION

BY
THOMAS RUSSELL GARTH

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in the
Faculty of Philosophy, Columbia University

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INTRODUCTION

A SURVEY OF THE SUBJECT OF MENTAL FATIGUE DURING CONTINUOUS EXERCISE OF A SINGLE FUNCTION

So far the investigations of various experimenters in the field of Mental Fatigue during continuous exercise show that continuous work under the lash of keen motivation reduces the efficiency only slightly, even though the subjects worked from one to two hours. Burgerstein, '*91*, found in an experiment with a large group of boys, aged eleven to thirteen years, in which the pupils performed easy examples in addition and multiplication in ten-minute periods with five-minute rests between, that the number of examples increased as the test proceeded, but there were more errors and corrections made by the pupils as the work went on.

Höpner, '*94*, undertaking to make the exercise more like that of the ordinary recitation, had a class of forty-six boys, of average age, nine years, write nineteen sentences from dictation. Each sentence was read aloud to the class once, then it was repeated once by a single pupil, next by the entire class, and then the pupils were required to write it from memory. The experiment extended over two hours. Höpner found that as the time proceeded, the pupils became less exact. Colloquial speech often took the place of the words in the dictated sentence. It is very evident that the materials for the experiment were not of equal difficulty. Neither were the pupils pressed to their utmost. (Offner & Whipple, *Mental Fatigue*, 57ff.)

Holmes, '*95*, had children from Grades Three to Eight add numbers for thirty-six minutes and broke the time into four periods of nine minutes each. There was an increase in errors of adding and copying. But when everything in the way of length of problem and speed is taken into consideration, the decrease in efficiency is not very great.

Cattell and Dolley found that simple reaction-time is not influenced by previous reactions to any great extent.

Thorndike describes an experiment of continuous mental work "of from three to eight hours" which "failed in the case of Dr. R. S. Woodworth, to produce any demonstrable diminution in efficiency." One part of the experiment was to mark the *e's* and *t's* in a book of 151 pages. This process continued for eight hours with less than nine minutes interruption. (See Thorndike's *Educational Psychology*, Vol. III, pp. 29–32.) Thorndike, '*11*, had fourteen students work from four to fourteen hours doing mental multiplication of three-place numbers by a like number. Six of these individuals took fifty-four per cent. more time for the end of the period on one day than they did at the beginning of the period for the preceding day. Six others had an average increase of fifty-nine per cent. For the other two, the increase was twenty-one per cent.

Arai, '*12*, had eleven students perform multiplication of two-place numbers mentally for two hours on two occasions. The intermissions were not the same for all participants in the experiment. The measure of efficiency was the time required for each example. It was found that the time was reduced. In performing the multiplications this means that there was an increase in efficiency.

Kraepelin worked out a method for continuous adding. A notebook is prepared with numbers printed in vertical columns. A bell is made to ring every five minutes and when the subject hears the bell he makes a mark. In this way the record is made according to the time worked. Kraepelin and his followers (Offner & Whipple, pp. 62–73) claim to have found certain interesting features in the work curve. These are practice, warming-up (Anlauf), swing (Anregung), spurt (Antrieb), initial spurt if it occurs at the beginning and 'spurt of change' if it occurs when a new kind of work begins, and 'terminal' spurt or end spurt (Schlussantrieb) when the subject realizes that he is nearing the end of the experiment or exercise.

Thorndike's absolutely fair treatment of the results and data of Kraepelin's work and of those who follow him brings into question the existence of such features in all work curves, and calls for further proof of their existence. He says: "Two hours or less of continuous exercise of a function at maximum efficiency produce a temporary negative effect . . . . of not over ten per cent. and in most functions still less than that."[1]

[1] *Educational Psychology*, Vol. III, pp. 68–69.

F. M. Phillips, *Journal of Educational Psychology*, 1916, had pupils of school grades of Fourth to Eighth work for ten minutes on arithmetic problems of addition, subtraction, multiplication and division. At the end of each minute the subject made a mark on the paper so that the experimenter knew what was done each minute by the subjects. In all the processes the best work was accomplished during the first minute. But the experimenter did not undertake to show whether there was fatigue or not. He found correlations between the first and last periods, in all the processes, ranging from .54 to .73, and says that the work of the first minute is less variable than that of later minutes, and that it is a fair index in all probability of one's ability in arithmetic.

W. S. Painter, *Journal of Educational Psychology*, 1916, did mental multiplication of four-place numbers after much mental and physical work which had resulted in a feeling of fatigue. He worked from 11 p. m. until 3:07 a. m. During the latter part of the experiment there was "a marked rise in time" per problem and the experimenter thinks that "the outcome suggests strongly that there exists a definite and relatively abruptly appearing point beyond which mental work becomes impossible." While the time in performing the last three multiplications is high, the errors are relatively few. Mr. Painter believes, however, that after this abrupt point all mental work, properly so called, was practically impossible.

CHAPTER I

THE ADMINISTRATION OF THE EXPERIMENT

The Subjects of the Experiment

There were seven hundred and eleven subjects in these experiments. Three hundred and sixty-eight of them were children in the Third and Fourth grades, and three hundred and forty-three of them were children of the Seventh and Eighth grades of the public schools of Virginia, all white children. A large part of the number was from the city schools of Richmond and Petersburg. The smaller part was from the Training School of the State Normal School at Farmville, Va., and the public schools of Farmville, Va. The boys and girls were about equally divided and their ages were those of the average for the grades in which the tests were given. There was absolutely no selecting of subjects, but the experimenter took the cases where he could get them. For this reason it may be taken as a fact that the results represent the work of the average child of these grades.

Materials Used

The materials used were the Thorndike addition sheets, such as those used by Kirby in the experiment described in his "Practice in the Case of School Children," Teachers College, Columbia University Contributions to Education, No. 53. There are seven different sheets. Each has forty-eight columns of one-place numbers, each column contains ten addends with the *1's* and the *0's* omitted, and each column is so arranged that any successive five of the columns are of a difficulty nearly, if not exactly, equal. These sheets were arranged in pads. For the Third and Fourth grades there were fourteen sheets, and for the Seventh and Eighth grade group there were twenty-one sheets in a pad. There was no possibility of a child's remembering the answers on a sheet which recurred, because there would be no way of identifying that sheet. Eye strain was reduced to a minimum because the type is so very large, and besides, the eyes got a rest every two minutes.

Conditions of the Experiment

Since the purpose of the experiment was to ascertain the behavior of the child mind during a recitation, both the materials and the conditions of the experiment were made as nearly like a recitation—or a drill recitation—as was possible under the circumstances. The materials were ordinary addition problems of one column, such as a teacher might use for a drill on adding and for improvement in number combinations. The experimenter gave all the tests in the presence of the regular teacher, and relied on his own teaching experience to put the child perfectly at his ease. There was nothing strained or strange about the experimenter's attitude but there was a constant endeavor to make the manner and atmosphere of the experiment entirely like that of an ordinary recitation-drill.

The subjects were told that the experimenter wished to find out who was the best adder in the class and that papers would be passed for this purpose, if the class was agreeable to it. The students were usually eager to try. They were told that the papers would be examined and that the name of the best adder would be announced later on by the teacher. The pads were then passed to the pupils, face downward. The experimenter used a stop watch. At a signal the pupils turned the pads face upward and added for two minutes when time was called. As quickly as possible with no intentional intermission except such as was necessary to keep the class together, the class received the instruction to "take the next sheet" and the signal "go" was given, when they again worked two minutes by the stop watch. The intermissions were only about ten seconds long. But in this time the subjects raised their eyes to the experimenter who tried to maintain a 'sympathetic' countenance such as would keep them working happily. Nothing more was said to the class by the experimenter after the initial motivation, excepting "take the next paper—get ready—go!" and after they had worked two minutes, "stop." The Third and Fourth grade pupils worked thus for twenty-eight minutes, and the Seventh and Eighth grade pupils for forty-two minutes.

The pads supplied a record of what was done every two minutes by each pupil. After the total time had passed the pads were collected. Each pad bore the name, age, and school and grade of the pupil on the back written by the pupil when he received the pad.

Method of Scoring

With fourteen sheets to every one of the 368 Third and Fourth grade pupils and 21 sheets to every 343 Seventh and Eighth grade pupils, the experimenter secured 12,355 papers. For the Third and Fourth grades there were fourteen periods of two minutes each and for the older group there were twenty-one periods of two minutes each. When these papers were examined, a record was made of what each child attempted each two minutes, and another record of the accuracy of each child for every two-minute period. If a child attempted six columns, he received a credit for that two minutes of six columns attempted. If only four of these were correct, he received credit on the accurate record sheet of only four columns. These two records, one of attempts and one of accurates, gave two tables, in the one case having fourteen columns 368 figures long, and in the other having twenty-one columns 343 figures long for attempts, and likewise the same for accurates. In short, we should say that the two groups of children—Third and Fourth grades and Seventh and Eighth grades—had two arrays of results, one for attempts and one for accurates.

If a child's record showed itself incomplete—nothing attempted after a certain time—except in the last period or so, his work was not counted in the experiment. One child had to stop because of a slight illness which she had when she came to school that morning. Another boy was convalescent from typhoid fever and had to stop in the midst of the experiment. If a lack-a-daisical attitude toward the experiment was evidenced by any great omission of effort in the written record, the material was thrown out. But it so happened that there was very little that had to be discarded. A child was given credit for what he did, both in quantity and quality.

Time of Day and Year

According to Heck's[1] experiments, the time of day makes little difference and so the experimenter made his tests at any time during the school day—early morning period, noon, and afternoon indiscriminately. Since there was a generous sampling of all times of the day, no one time of the day with its fatigue could assert itself

[1] W. H. Heck, *A Study of Mental Fatigue*, 1913.

to any great extent. The test was intended to show what happens during a recitation at any time of the day.

The time of the year was in the fall, in the winter, and in the spring. One class took its test when the snow was falling on the ground, a large part of the groups took their tests in November, and some of the classes worked with the windows open to the spring breeze. The tests were given during the time elapsing between December, 1913, and November, 1914.

CHAPTER II

ATTEMPTED AND ACCURATE PERFORMANCE

The Group Curves

Four curves are given herewith, plotted from the tables which accompany them. There are two work or attempts curves, one for Third and Fourth grades and one for Seventh and Eighth grades; and there are two accurates curves, respectively, for Third and Fourth grades and for Seventh and Eighth grades. In the Third and Fourth grade group there were 368 children participating and

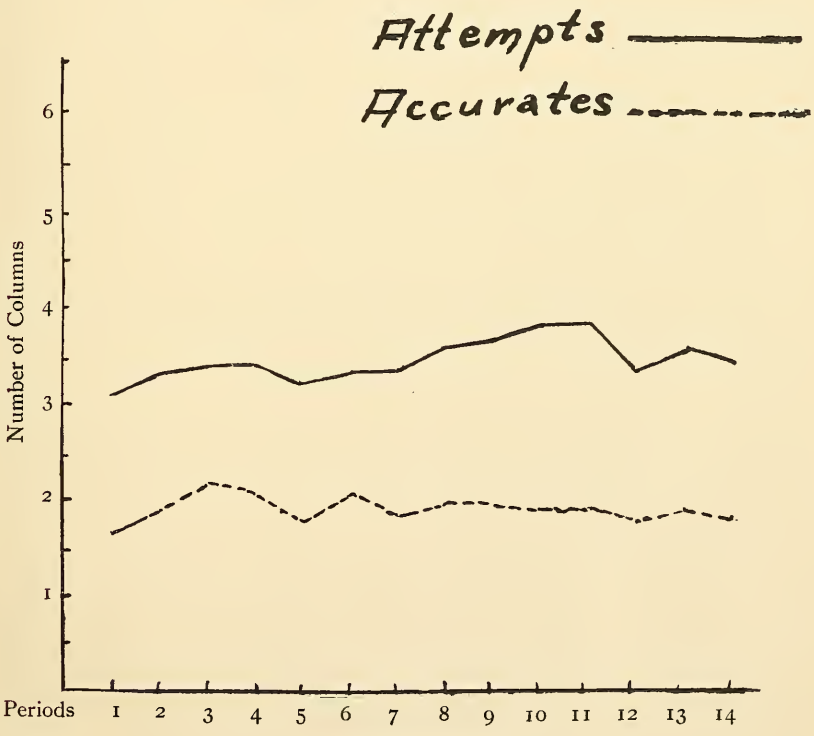


Fig. 1. Third and Fourth Grades

in the Seventh and Eighth grade group there were 343 who took part. The Third and Fourth grade group worked for twenty-eight minutes actually. This time was divided into fourteen periods of two minutes each. The horizontal line represents the time, in periods, during which the groups worked. The vertical line indicates the number of columns either attempted or accurately done. In the case of the work curve this vertical line indicates columns attempted whether right or wrong; in the case of the accuracy curve

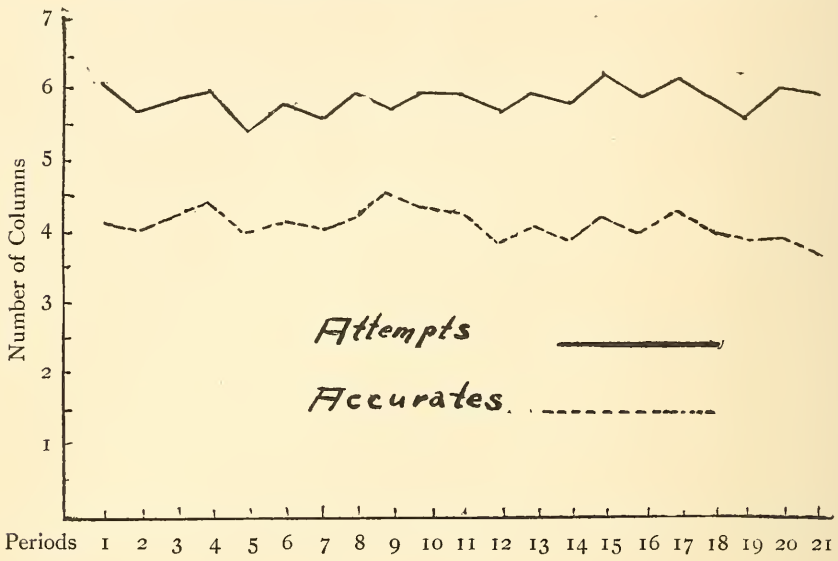


Fig. 2. Seventh and Eighth Grades

the vertical scale stands for columns correctly added. Every point on the curve was ascertained by finding the average attained by the group for that identical two-minute period of work. We shall call these the group curves.

These curves are given here for the sake of presenting the facts derived by the experiment. These are mere gross results. The attempts and accuracy curves derived herewith do not afford a true idea of measurement of fatigue of the individuals during the work period of twenty-eight minutes for the Third and Fourth grade group nor for the forty-two minutes during which the Seventh and Eighth grade group worked. If instead of giving the average

we gave the quantity obtained by the group, this would emphasize the production by the group as a whole. These first curves give the changes in the group without regard to the changes of the individual. The curves following these will give the average of the individual curves and thus will regard individual changes.

TABLE I

Attempts of Third and Fourth Grades—Absolute Measures

Columns attempted for each period of two minutes

Period	*Average*	*A. D.*	*P. E.*
1	3.15	1.20	.058
2	3.35	1.17	.051
3	3.47	1.22	.053
4	3.48	1.34	.059
5	3.27	1.17	.051
6	3.37	1.25	.055
7	3.45	1.34	.059
8	3.54	1.33	.058
9	3.55	1.38	.061
10	3.57	1.39	.061
11	3.49	1.31	.057
12	3.28	1.31	.057
13	3.51	1.41	.062
14	3.38	1.36	.060

TABLE II

Accurates of Third and Fourth Grades—Absolute Measures

Columns accurate for each period of two minutes

Period	*Average*	*A. D.*	*P. E.*
1	1.70	1.27	.056
2	1.93	1.1	.048
3	2.11	1.32	.058
4	2.06	1.28	.056
5	1.88	1.16	.051
6	2.07	1.24	.054
7	1.86	1.21	.053
8	1.99	1.16	.051
9	1.98	1.34	.059
10	1.96	1.30	.057
11	1.94	1.27	.056
12	1.79	1.30	.057
13	1.82	1.21	.053
14	1.78	1.30	.057

TABLE III

Attempts of Seventh and Eighth Grades—Absolute Measures

Columns attempted for each period of two minutes

Period	*Average*	*A. D.*	*P. E.*
1	6.02	1.67	.075
2	5.62	1.39	.062
3	5.77	1.5	.068
4	5.92	1.61	.072
5	5.46	1.51	.068
6	5.78	1.55	.07
7	5.64	1.53	.069
8	5.81	1.69	.076
9	5.68	1.77	.079
10	5.98	1.52	.068
11	5.94	1.56	.07
12	5.65	1.69	.076
13	5.99	1.5	.068
14	5.83	1.68	.076
15	6.23	1.95	.088
16	5.91	1.52	.068
17	6.11	1.67	.075
18	5.95	1.84	.083
19	5.61	1.7	.077
20	6.02	1.78	.08
21	5.99	1.91	.086

TABLE IV

Accurates of Seventh and Eighth Grades—Absolute Measures

Columns accurate for each period of two minutes

Period	*Average*	*A. D.*	*P. E.*
1	4.22	1.85	.083
2	4.04	1.63	.074
3	4.19	1.50	.068
4	4.30	1.83	.082
5	4.01	1.57	.061
6	4.18	1.94	.087
7	4.03	1.59	.072
8	4.22	1.64	.074
9	4.64	1.65	.073

TABLE IV (*Continued*)

Accurates of Seventh and Eighth Grades—Absolute Measures

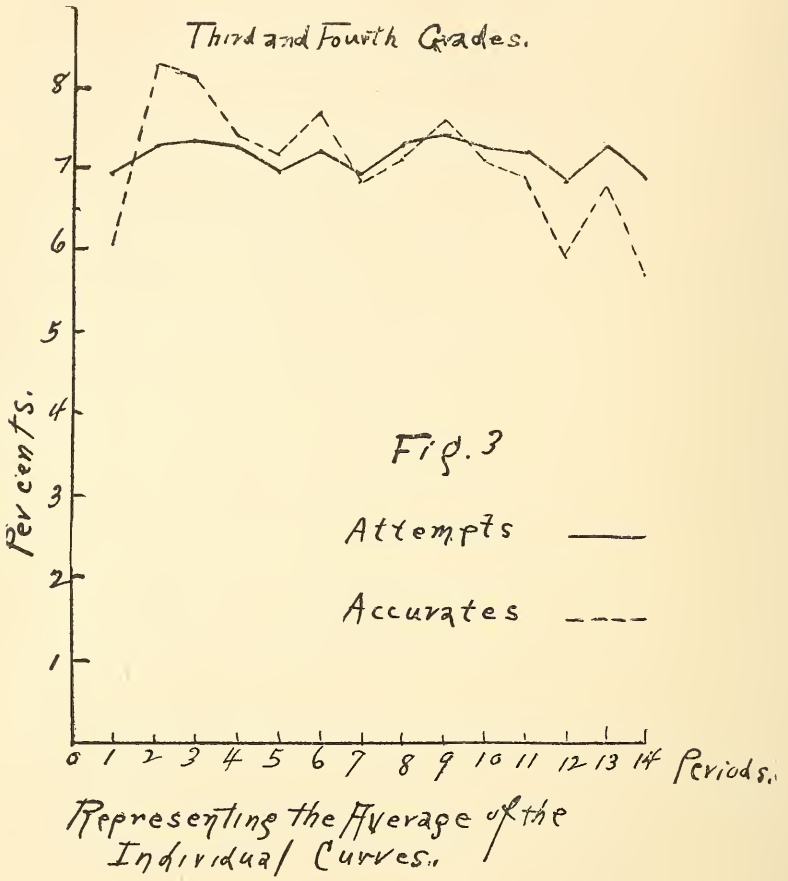
Columns accurate for each period of two minutes

Period	*Average*	*A. D.*	*P. E.*
10	4.31	1.68	.075
11	4.29	1.71	.076
12	3.89	1.76	.079
13	4.19	1.74	.078
14	3.88	1.76	.079
15	4.24	1.91	.085
16	4.03	1.71	.077
17	4.26	1.72	.077
18	4.02	1.72	.077
19	3.88	1.77	.080
20	3.93	1.83	.082
21	3.74	1.82	.081

Average of the Individual Curves

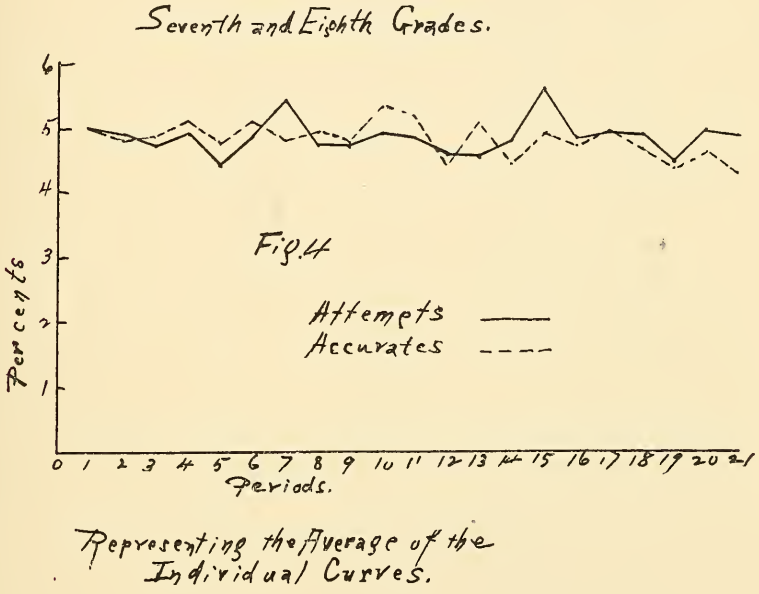
The object in a study of this kind is to give every individual, whether his work is slow or fast, the same influence upon the final average. It is not fair to the poor or weak individual to put his small effort on a basis of absolute terms along with the effort of the better workers similarly expressed. The former may fatigue while the latter gain and a mean between them show essentially the curve of the better worker. The original data of the preceding tables fail to represent truly the facts which we wish to bring out because, as intimated above, the rapid workers with their larger measures of performance entirely swamp the changes that may occur in the work of the slower ones. But if everyone's work for the successive periods is reduced to percentages of his total, the slow count the same as do the fast workers in determining the general work curve. For this reason we have derived percentage curves which represent more truly than the foregoing curves the central tendency of the whole group with regard to fatigue. With the absolute numbers we can examine the question of fatigue from the standpoint of one interested in the total product of a group, whereas by the personal curves we disregard the total products and examine changes in the

working ability of the individuals, and the general tendency of these individual changes.



The curves shown herewith (Figures 3 and 4) are, however, only slightly different in form from the preceding curves. Every number in the distribution is a percentage of the total accomplished during the entire period of work for each installment of time for each child. A child did so much every two minutes for twenty-eight and forty-two minutes in the Third and Fourth grade group and the Seventh and Eighth grade group respectively. Percentages of these dis-

tributions were determined by dividing each two minutes' work by the total amount added during the whole period of work by each individual. This gave fourteen per cents. in the younger group for each child, and twenty-one per cents. in the Seventh and Eighth grade group for each child of that group. The fourteen average per cents. for the Third and Fourth grade group were obtained,



Representing the Average of the Individual Curves.

and the twenty-one average per cents. for the Seventh and Eighth grade group likewise were obtained, and these averages give the accompanying curves. Every position on the curves indicates the central tendency of the relative value in attempts and accuracy of the groups for any two minutes of the time of working; and this should be taken as meaning the comparative working ability at that period. The attempts curve does not signify a pure mental performance, while the accurates curve here expressed in percentages does represent a more nearly pure mental process.

TABLE V

Data for the Curves Representing the Average of the Individual Curves—Third and Fourth Grades

Columns attempted in each period of two minutes, expressed as a per cent. of the total number attempted

Period	*Range Per Cent.*	*Average Per Cent.*	*A. D.*	*P. E.*
1	0–50	6.97	1.94	.09
2	0–25	7.24	1.52	.08
3	0–17	7.30	1.37	.06
4	0–17	7.24	1.40	.06
5	0–25	6.97	1.32	.06
6	0–14	7.20	1.32	.06
7	0–18	6.97	1.35	.06
8	0–25	7.29	1.43	.06
9	0–17	7.34	1.39	.06
10	0–12	7.28	1.44	.06
11	0–14	7.27	1.34	.06
12	0–13	6.78	1.47	.07
13	0–13	7.25	1.31	.05
14	0–16	6.90	1.62	.07
		100.00		

TABLE VI

Data for the Curves Representing the Average of the Individual Curves—Third and Fourth Grades

Columns accurate for each period of two minutes, in per cent. of total columns accurate

Period	*Range Per Cent.*	*Average Per Cent.*	*A. D.*	*P. E.*
1	0–100	6.13	4.53	.20
2	0–100	8.34	4.87	.22
3	0–50	8.16	4.34	.21
4	0–34	7.53	4.03	.13
5	0–100	7.27	4.20	.19
6	0–50	7.76	4.10	.18
7	0–100	6.94	4.01	.18
8	0–28	7.24	4.05	.18
9	0–100	7.62	4.57	.21
10	0–50	7.16	4.31	.19
11	0–29	7.06	4.27	.19
12	0–30	6.10	4.07	.18
13	0–34	6.89	4.30	.19
14	0–40	5.80	3.75	.17
		100.00		

TABLE VII

Data for the Curve Representing the Average of the Individual Curves—Seventh and Eighth Grades

Columns attempted for each period of two minutes, in per cent. of total columns attempted

Period	*Range Per Cent.*	*Average Per Cent.*	*A. D.*	*P. E.*
1	2–10	5.00	1.01	.05
2	0–10	4.86	.79	.04
3	0–9	4.67	.77	.04
4	2–10	4.79	.69	.03
5	2–8	4.37	.75	.03
6	3–8	4.73	.72	.03
7	2–8	5.30	1.04	.05
8	1–11	4.63	.79	.04
9	2–9	4.59	.75	.03
10	3–8	4.86	.75	.03
11	1–8	4.80	.68	.03
12	2–10	4.43	.74	.03
13	1–16	4.42	.88	.04
14	2–7	4.68	.83	.04
15	1–9	5.58	1.11	.05
16	1–9	4.74	.62	.03
17	0–9	4.80	.71	.03
18	0–9	4.75	.90	.04
19	0–8	4.40	.86	.04
20	0–9	4.84	.91	.04
21	0–11	4.76	.90	.04
		100.00		

TABLE VIII

Data for the Curve Representing the Average of the Individual Curves—Seventh and Eighth Grades

Columns accurate for each period of two minutes, in per cent. of total columns accurate

Period	*Range Per Cent.*	*Average Per Cent.*	*A. D.*	*P. E.*
1	0–20	5.00	1.74	.08
2	0–14	4.85	1.51	.07
3	0–30	4.85	1.51	.07
4	0–27	5.01	1.37	.06
5	0–11	4.50	1.37	.06
6	0–15	5.03	1.33	.06
7	0–20	4.70	1.46	.07
8	0–12	4.83	1.37	.06

TABLE VIII (*Continued*)

Period	*Range Per Cent.*	*Average Per Cent.*	*A. D.*	*P. E.*
9	0–12	4.73	1.30	.06
10	0–22	5.15	1.48	.07
11	0–17	5.10	1.28	.06
12	0–12	4.34	1.45	.07
13	0–12	5.03	1.39	.06
14	0–10	4.37	1.51	.07
15	0–13	4.90	1.48	.07
16	0–13	4.74	1.55	.07
17	0–11	4.84	1.42	.06
18	0–23	4.73	1.23	.06
19	0–13	4.40	1.35	.06
20	0–10	4.50	1.53	.07
21	0–12	4.30	1.69	.08
		100.00		

The Similarity between Attempts and Accurate Performance

The question arises as to whether the general tendencies of the attempts curve and the accurates curve are the same or different. Possibly the best evidence that they are rather similar is to be gotten by merely examining the curves themselves. (See Figures 1 to 4.) While the accuracy and attempts curves are not parallel they are decidedly similar in their general tendencies. To give further evidence of this similarity, correlations, though not as good evidence as the curves themselves, were gotten between the two kinds of curves.

The correlations between the Averages of Tables I and II, taken as a whole, is +.487. The correlation for the Averages of Tables III and IV, taken as a whole, is +.29. For Tables V and VI, and for VII and VIII, the correlations are respectively +.58 and +.32. The older group has the lower correlations. The curves derived from the per cents.—the curves called the average of the individual curves—give higher correlations than the curves derived from absolute measures.

Therefore, numbers for attempts and numbers for accurates show a positive tendency to vary together. They do not vary inversely, if we take the curves as a whole, though doubtless there are corresponding parts of the curves that do. Some of these divergencies will appear as we proceed in our examination of the curves.

TABLE IX

Absolute Measures

	Third and Fourth Grades		*Seventh and Eighth Grades*	
	Attempts	*Accuracy*	*Attempts*	*Accuracy*
I. Maximum period	Tenth	Third	Fifteenth	Ninth
II. Minimum period	First	First	Fifth	Second
III. Average first three periods	3.32 cols.	1.91 cols.	5.80 cols.	4.15 cols.
IV. Average last three periods	3.39 cols.	1.79 cols.	5.87 cols.	3.85 cols.
V. Per cent. loss or gain III and IV	2.0 gain	6.2 loss	1.0 gain	7.0 loss
VI. Average part one (or first third of curve)	3.36 cols.	1.95 cols.	5.74 cols.	4.14 cols.
VII. Average part two (or second third of curve)	3.44 cols.	1.96 cols.	5.84 cols.	4.20 cols.
VIII. Average part three (or third third of curve)	3.45 cols.	1.86 cols.	5.97 cols.	4.01 cols.
IX. VI as base	1.00	1.00	1.00	1.00
VII : VI	1.023	1.005	1.017	1.014
VIII : VI	1.026	.95	1.04	.97

TABLE X

From the Data of the Curves Representing the Average of the Individual Curves

	Third and Fourth Grades		*Seventh and Eighth Grades*	
	Attempts	*Accuracy*	*Attempts*	*Accuracy*
I. Maximum period	Ninth	Second	Fifteenth	Tenth
II. Minimum period	Twelfth	Four-teenth	Fifth	Twenty-first
III. Average first three periods	7.17%	7.54%	4.85%	4.90%
IV. Average last three periods	6.98%	6.26%	4.67%	4.40%
V. Per cent. loss or gain	3.0 loss	17.0 loss	4.0 loss	10.2 loss
VI. Average part one (or first third of curve)	7.19%	7.54%	4.82%	4.85%
VII. Average part two (or second third of curve)	7.15%	7.37%	4.63%	4.79%
VIII. Average part three (or third third of curve)	7.10%	6.60%	4.84%	4.63%
IX. VI as base	1.00	1.00	1.00	1.00
VII : VI	.99	.98	.96	.99
VIII : VI	.98	.88	1.004	.95

The accompanying tables, Nos. IX and X, give some of the data to be derived from the curves. Table IX is a summary of data gotten from the absolute measures and Table X is a similar summary gotten from the percentages. Using these data of the tables we shall discuss various factors which may influence the curves, as fatigue, warming-up, initial-spurt, practice, and end-spurt. Accordingly, the discussion will be from the two standpoints spoken of above, that of one interested in the total product and that of one interested in changes in working ability.

Maximum and Minimum Points on the Curve

The maximum point on a curve of continuous performance at least makes a good landmark for a preliminary survey of fatigue in that curve. Being the high point in the curve, the line must recede from it, be it ever so gradually, and this negative effect must be due to fatigue—granting that the workers are serious in their performance. The minimum must mean, if we still grant the seriousness of the workers, one of several things, *i.e.*, if it occurs before the maximum it must signify a lack of proper adaptation or a warming-up, if it follows the maximum it must be due to fatigue (from which there may be recovery). The maximum point in the curve does not tell us when fatigue begins to work in the performance; but it makes the presence of fatigue evident by the falling away that follows it. Consequently, it makes a good point at which to begin the discussion of fatigue.

If the decline in the curve following the maximum was not due to fatigue, it must have been due to ennui or loss of interest. It might better be said, shifting of interest. Now, because of the original strong motivation through appeal to the competitive impulse and because no other interest was permitted to arise in as far as lay within the power of the experimenter to prevent it—for the subjects really had no time for mind wandering, and were kept steadily at their tasks—for this reason the writer believes there was very little shifting of interest other than such as was due to the distraction accompanying fatigue.

For the group curves obtained from the absolute measures (See Table IX) the maximum of attempts was reached, in the case of the Third and Fourth grade group, in the tenth period or between the eighteenth and twentieth minutes. The maximum of accurates

occurred in the third period or at about the sixth minute. As to the minima, they occur before the maxima and indicate lack of adaptation in consequence. The foregoing facts indicate that the maximum was reached earlier by fourteen minutes in the accurates curve than in the attempts curve. For the similar curves of the Seventh and Eighth grade group the same observation is made that the maximum of accurates preceded that of attempts by ten minutes. The older group showed more persistence, as the maximum of accurates occurred in the ninth period or about the eighteenth minute and that of attempts in the fifteenth period, or between the twenty-eighth and thirtieth minutes. Oehrn found that when his adult subjects added continuously for two hours, they attained their maximal speed about twenty-eight minutes from the start. (Whipple, *Manual of Mental and Physical Tests*, 335.)

From the curves representing the average of the individual curves the same thing is true as to the maximal attainment of accuracy occurring before that of work in attempts. The minima of accuracy since they are found following the maximal points in both groups register fatigue. (See Table X for the data from which these observations are made.)

The two standpoints—interest in total work and in changes in working ability—make practically no difference as to the time of occurrence of the maxima.

The general conclusions to be drawn here are then:

1. Fatigue is made evident in a curve of this sort by the line falling away from the maximum.

2. Workers attain their maximum of accuracy before they attain that point in attempting.

3. In both quantity and quality of work, the younger children reach their maximum as a group before the older ones, and therefore notably fatigue sooner.

4. The occurrence of the minimum after the maximum is in consequence of fatigue and not from lack of adaptation.

Fatigue

If we wish to find fatigue in an attempt or accurate performance curve we look for it certainly at the end of the performance because, though it may have been a factor nearly all along the way, it would here be more in evidence than elsewhere. Fatigue is made evident

by a falling away from a previous performance. Now we must have something to measure by as well as something to measure. The basis of measurement or comparison taken here is the average of the first three periods of work. By taking our standard at the beginning we avoid practice and fatigue effects.

The question of novelty of stimulus is of slight importance here since the subjects were rather familiar with such material as that given in the experiment.

That part of the data of Tables IX and X given as III and IV, 'average of first three' and 'average of last three', forms the measures which are compared for the purpose of ascertaining the fatigue at the end of the total time of working. 'Average first three' signifies the average of the first three periods of two minutes each, and the average of the last three periods of two minutes each is signified by 'last three'. The facts under III give the basis of comparison. V, 'per cent. of loss or gain', indicates the results of the comparisons for the various curves. The results show that in absolute measures attempts gained slightly and accurate performance lost. The gain for attempts of Third and Fourth grade group was 2 per cent., for Seventh and Eighth grades, 1 per cent. The fatigue in accurates of Third and Fourth grade groups was 6.2 per cent. and for Seventh and Eighth grades was 7 per cent.

The result of similar comparisons in Average of Individual Curves made after the same fashion as that of Absolute Measures shows there was fatigue in both attempts and accurates in both the younger and the older groups. (See Table X.) Fatigue in attempts for the former was 3 per cent., for the latter was 4 per cent., but the accurates suffered a 17 per cent. loss for the younger group and 10.2 per cent. for the older.

If the writer's contention is accepted that the curves representing the Average of the Individual Curves best represent the facts, we may conclude that attempts and accurates alike suffer fatigue in an exercise of this sort.

General Form of the Curves

We first present Figures 1-a, 2-a, 3-a and 4-a, which are "smoothed" from Figures 1, 2, 3 and 4 respectively, by taking the averages of the first and second periods, of the third and fourth, etc.; *i. e.*, the two-minute periods of the experiment are here combined into units

of four minutes each. Some of the chance irregularities of the original curves being thus removed, the general course of the curves can be better seen.

From Figures 1-a and 2-a it appears that the working power of the group as a whole remained very nearly constant during the experi-

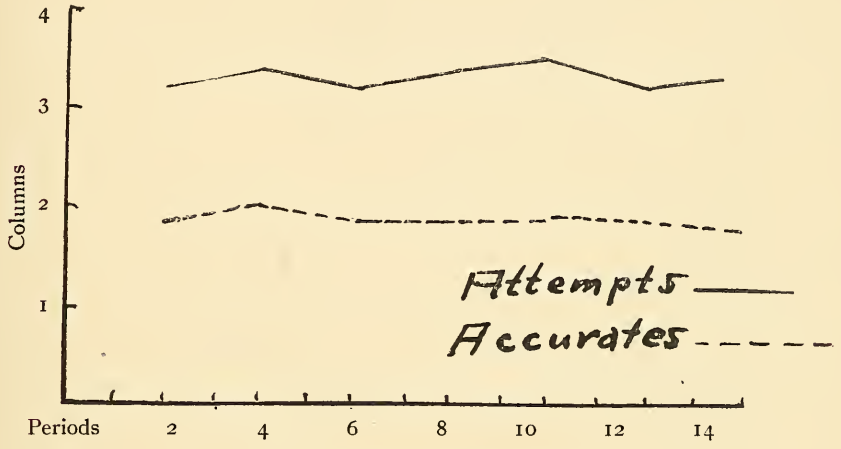


Fig. 1-a. Third and Fourth Grades

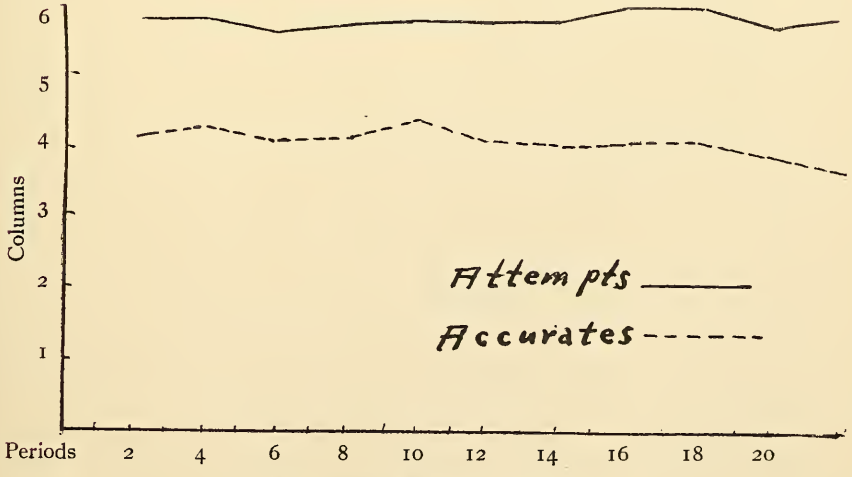


Fig. 2-a. Seventh and Eighth Grades

ment, though there is a slight but definite descent in the curves for accurate work, and this decline begins earlier in the younger than in the older group.

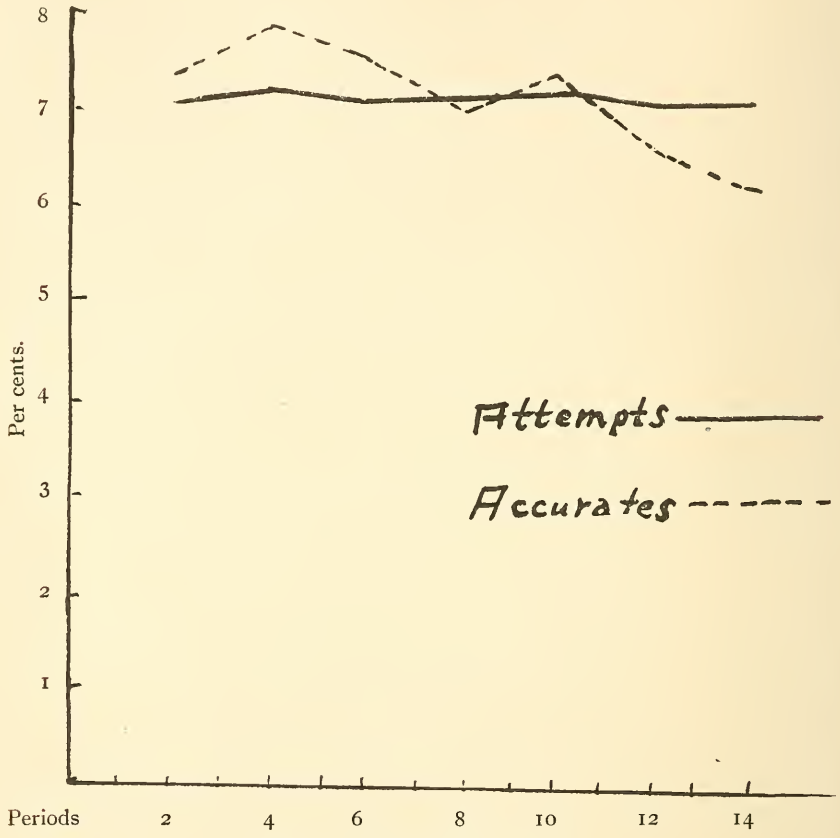


Fig. 3-a. Third and Fourth Grades

We next divide the curves into three parts, as nearly as possible thirds in the Third and Fourth group, and exact thirds in the Seventh and Eighth grade group. The former group is divided thus—four periods in the first part and five periods in the second and third parts. The fact that there were fourteen periods makes this unequal division necessary. The Seventh and Eighth grade group curve is divided into three parts, each containing seven

periods. In the accompanying table, No. IX, for Group Curves, if the reader will refer to VI, VII, and VIII, he will find the averages and ratios to which we are going to refer. In the case of the Third and Fourth grade group the average amount attempted each two minutes for this part of the curve (eight minutes) was 3.36 columns, for the second part or ten minutes following the first division of time, the average was 3.44 columns and for the last ten minutes of

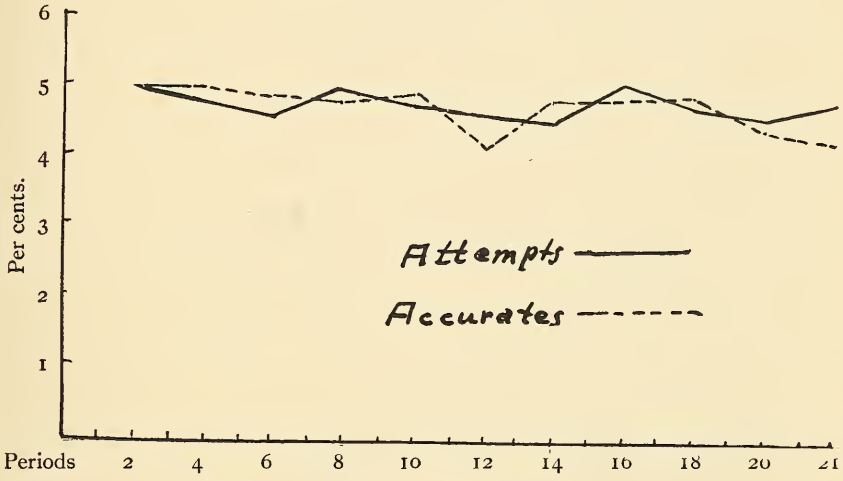


Fig. 4-a. Seventh and Eighth Grades

work 3.45 columns were attempted for two minutes on the average. This shows a steady rise, in the ratios of 1.00, 1.023, and 1.026.

In the case of the Seventh and Eighth grade group the three equal parts of the whole forty-two minutes show thus an average for the first fourteen minutes of 5.74 columns attempted, for the second fourteen minutes 5.84 columns were attempted and for the last fourteen minutes of time an average of 5.97 columns were attempted for two minutes. This likewise shows a rise. The ratios are 1.00, 1.017, and 1.04. These rises are so slight that too much significance should not be given them.

In the case of accurates, the first eight minutes in the Third and Fourth grade groups shows an average of 1.95 columns correctly solved for each two minutes. The next ten minutes of the total time produced an average of 1.96 columns correctly solved for each

two minutes and the last ten minutes shows an average of 1.86 columns per period of two minutes each, correctly added. The ratios are 1.00, 1.005, and .95. With the older groups the three averages are 4.14, 4.20, and 4.01. The ratios are 1.00, 1.014, and .97.

The accurate performance curves here are seen to rise in the middle and decline at the end, but the attempts curves rise steadily in both groups. Practice was helping out very much in this, for them, quasi-automatic process.

Therefore, taking the group curves in three parts as we have just done shows (1) slight practice effect in the attempts process in both groups, (2) slight practice effect in the accuracy curve in the second third of the partition of the curve but fatigue in the last third in both groups, all of which holds alone for the absolute measures.

The above facts hold for the curves derived from absolute measures, but the corresponding parts of the curves showing average of the individual curves are somewhat different in their relations.

The numbers in the curves are these (See Table X):

Third and Fourth Grades	*Attempts*
First Third, as base	1.00
Second third : first third	.99
Third third : first third	.98

Third and Fourth Grades	*Accuracy*
First third as base	1.00
Second third : first third	.98
Third third : first third	.88

Seventh and Eighth Grades	*Attempts*
First third as base	1.00
Second third : first third	.96
Third third : first third	1.004

Seventh and Eighth Grades	*Accuracy*
First third as base	1.00
Second third : first third	.99
Third third : first third	.95

The result is accordingly that the average individual falls off slightly in work attempted, with some recovery toward the end in the case of the older individuals (Seventh and Eighth grades); while in accurate work, the average individual shows a greater falling off, without recovery, and the falling off is greater in the younger child. The conclusion from this form of comparison is in general harmony with that derived from the immediately preceding comparison and with that derived from inspection of the curves.

'Warming-up' and 'Initial-Spurt'

We now proceed to examine these curves to see if there is 'Warming-up' or 'Initial-spurt'. "The best definition of 'Warming-up' as an objective act is that part of an increase of efficiency during the first twenty minutes (or some other assigned early portion) of a work period, which is abolished by a moderate rest, say of sixty minutes." Thorndike, *Educational Psychology*, vol. III, p. 66. "'Initial-spurt', if a real fact, will be found in an examination of the work, minute by minute, of the first quarter of an hour." *Ibid.*, p. 48. A sudden rise at the start will indicate 'Initial-spurt'.

The graphic representation of the attempted and accurate work affords perhaps the best objective material for determining the presence or absence of these phenomena in the performance under discussion, and so we call attention to the curves given in Figures 1 and 2, and derived from Tables I, II, III, and IV for Absolute Measures, and Figures 3 and 4 derived from Tables V, VI, VII, and VIII. The curves for the younger group show a fairly steady rise in the first six minutes in both attempts and accurate performances in all the curves representing them, and from this evidence we may say there was 'Warming-up' for them. If it should be objected that these rises were merely incidental and due to chance, we may resort to the tables showing the averages and their probable errors. By taking the difference between the averages of the successive periods, first and second, second and third, and bringing these into relation to the square root of the sum of their respective squared P. E.'s, we may determine the reliability of these differences. We give here these reliabilities. The chances that the gain of period two over period one, in Table I, is reliable are 91 out of 100; for period three over period two they are 75 out of 100. This 'Warming-up' holds there for attempts in the Third and Fourth grades. As to accurates the case is even better, for the chances that the gain of period two over period one, in Table II, is reliable are 96 out of 100, and of the gain of period three over period two are 92 out of 100, in the Third and Fourth grades. This holds for the Absolute Measures.

In the case of the curves representing the Average of the Individual Curves, we have the following facts as to 'Warming-up'. The chances that the gain of period two over period one is a true

one are 87 out of 100. The chances that the gain of the third period over the second period (Table V) is significant are not so great, being only 32 out of 100 cases. In Table VI the gain of second period over the first, has a reliability of 84 cases out of 100. But there is no gain of the third period over the second, so that the 'Warming-up' disappeared by the time the sixth minute was reached.

Therefore, we may conclude, since the rise in the curves is found to be more or less reliable, that the 'Warming-up' found at the beginning of the work of the younger group was a real fact.

Now if there is here found in the younger group positive evidence of 'Warming-up', the fact excludes the possibility of 'Initial-spurt', which is in a measure the opposite of 'Warming-up'.

The curves for the older group do not show a 'Warming-up 'effect upon inspection. The rather sudden slight rise in the first period, followed by a fall in the second period, on the face of things indicates 'Initial-spurt' in both attempted and accurate performance. If we examine periods one and two (Table III) and take into account their probable errors we find that the loss of the second period over the first, has a reliability as a change of 98 out of 100 cases. This is for attempts. For accurates we take periods one and two, with their probable errors (Table IV), and find that the loss here is a difference whose reliability is 82 chances out of 100. These tables give the data for the Absolute Measures. The data for the curves representing the average of the Individual curves is found in Tables VII and VIII for attempts and accurates respectively. For attempts the chances that the change in going from the first period to the second is a real fact are 88 out of 100 cases and for accurate performances are 74 out of 100. The indications of 'Initial-spurt' are found thus to be fairly reliable.

Therefore, in so far as these data are concerned, the older children experienced an 'Initial-spurt' which is opposite to the 'Warming-up' experienced by the younger ones.

'End-Spurt'

Since the experiment was planned so as to avoid anything like 'End-spurt' and let fatigue do its worst at the end, it hardly seems worth while to examine these data with the intention of finding 'End-spurt', which, according to Kraepelin and others, is a sudden

rise in efficiency, due to a knowledge on the part of the subjects of the approaching end of the task.

The experimenter here did all in his power to conceal the approaching end of the test. For this reason we do not think these data should be examined with the end in view of finding 'End-spurt'.

The writer has a conviction that interference operated at the beginning of the test because of the strong motivation. The desire to excel must have acted as a hindrance at this part of the game inasmuch as it probably acted as a distraction from letting the necessary associations play freely. Consequently, it would lower the score especially of accuracy. It also probably played strongly toward the end of the test and may account for some of the fatigue.

CHAPTER III

INDIVIDUAL DIFFERENCES

Individual Curves

In the preceding chapter, individual differences were eliminated and the general tendencies to fatigue, etc., of large groups alone considered. If we take the records of individuals, we shall, of course, find every variety of curve. Some do most of their work at the start and show fatigue considerably towards the end, some work steadily from start to finish, and some improve all the way. Such differences may be due to slight intercurrent causes, and are not, in the lack of further evidence, to be regarded as indicative of different types of workers. It may be well, however, to present the records of a few individuals, selected more or less at random from among those whose total performance was small, medium, or great. The data given in Table XI show for each individual the per cent. of his total accomplishment done in each successive period of two minutes. Data are given for twenty individuals in all, ten from the Third and Fourth grades and ten from the Seventh and Eighth, five in each group showing the results for attempts and five for accurate work.

For example, Case 22 from the Third and Fourth grade group attempted twenty-one columns in all, twenty-three per cent. of these being attempted in the first three periods, sixty-one per cent. in the first half of the time, and only nineteen per cent. in the last three periods—showing thus a considerable falling off in energy towards the end. Case 367, on the contrary, attempted only fourteen per cent. of his total of one hundred and ninety-one columns in the first three periods, forty-six per cent. in the first half of the time, and twenty-four per cent. in the last three periods.

These differences are found all along the way in going from the poorest total performance to the best. Some poor in total performance are steady workers. Some enthusiastic workers at the start do not do so well at the end, but do well in total performance. Those workers who profit by practice are found among the poorer

workers as well as among the better workers as determined by total performances.

TABLE XI

Showing Changes in Working Ability—Third and Fourth Grades
Per Cent. of Columns Attempted

Period	*Case 22* *Per Cent.*		*Case 138* *Per Cent.*		*Case 276* *Per Cent.*		*Case 344* *Per Cent.*		*Case 367* *Per Cent.*	
1	14		5		5		8		1	
2	14		8		5		6		5	
3	5		10		5		8		8	
		33		23		15		22		14
4	0		8		5		7		8	
5	14		7		7		7		8	
6	9		10		7		6		8	
7	5		5		5		8		8	
		61		53		39		50		46
8	6		7		9		8		8	
9	5		8		8		6		8	
10	5		5		8		8		8	
11	5		8		10		8		2	
12	5		5		10		8		8	
13	5		7		8		6		8	
14	9		7		8		6		8	
	100	19	100	19	100	26	100	20	100	24
Average per cent.	7.1		7.1		7.1		7.1		7.1	
Total columns attempted	21		40		60		79		191	

TABLE XI (*Continued*)

Third and Fourth Grades—Per Cent. of Total Accurate Columns

Period	*Case 142 Per Cent.*		*Case 246 Per Cent.*		*Case 329 Per Cent.*		*Case 350 Per Cent.*		*Case 368 Per Cent.*	
1	26		15		8		7		4	
2	16		15		8		7		7	
3	11		9		6		9		7	
	53		39		22		23		18	
4	11		9		10		8		7	
5	0		3		5		6		7	
6	0		0		8		8		7	
7	10		9		8		6		9	
	74		60		53		51		48	
8	5		15		8		7		6	
9	11		4		5		6		9	
10	0		12		8		8		8	
11	5		0		6		5		6	
12	0		0		6		9		8	
13	5		9		8		7		8	
14	0		0		8		7		7	
	100	5	100	9	100	22	100	23	100	23
Average per cent.	7.1		7.1		7.1		7.1		7.1	
Total accurate columns	19		33		50		60		110	

TABLE XI (*Continued*)

Seventh and Eighth Grades—Per Cent. of Columns Attempted

Period	*Case 32* *Per Cent.*		*Case 52* *Per Cent.*		*Case 215* *Per Cent.*		*Case 298* *Per Cent.*		*Case 336* *Per Cent.*	
1	8		4		3		4		5	
2	6		6		2		5		4	
3	5		3		3		5		4	
		19		13		8		14		13
4	5		3		3		6		5	
5	6		6		2		4		5	
6	6		6		1		5		4	
7	5		4		3		6		4	
8	5		6		3		6		4	
9	6		4		4		4		4	
10	4		3		4		5		4	
11	4		3		6		5		4	
		60		48		34		55		47
12	3		6		7		4		5	
13	6		6		8		4		4	
14	4		4		8		5		4	
15	5		3		6		4		4	
16	5		3		6		4		4	
17	4		6		6		5		4	
18	3		6		6		4		4	
19	3		6		5		4		4	
20	3		6		5		5		4	
21	3		6		4		6		4	
	100	9	100	18	100	14	100	15	100	12
Average per cent.	4.7		4.7		4.7		4.7		4.7	
Total columns attempted	77		89		133		160		223	

TABLE XI (*Continued*)

Seventh and Eighth Grades—Per Cent. of Total Accurate Columns

Period	*Case 20* *Per Cent.*		*Case 64* *Per Cent.*		*Case 208* *Per Cent.*		*Case 304* *Per Cent.*		*Case 343* *Per Cent.*	
1	10		10		2		5		4	
2	10		10		5		5		4	
3	12		4		2		5		5	
		32		24		9		15		13
4	7		6		3		5		6	
5	5		6		6		4		4	
6	3		6		6		5		5	
7	5		4		3		4		4	
8	7		8		6		5		4	
9	0		0		3		4		5	
10	3		6		6		5		4	
		62		58		42		47		45
11	5		4		6		4		6	
12	0		4		6		5		4	
13	3		5		3		5		6	
14	3		3		5		5		5	
15	3		4		5		5		5	
16	0		2		6		5		4	
17	4		4		5		5		5	
18	5		2		5		4		6	
19	3		4		5		5		6	
20	5		6		6		5		4	
21	7		4		6		6		4	
	100	15	100	14	100	17	100	16	100	14
Average per cent.	4.7		4.7		4.7		4.7		4.7	
Total of accurate columns	40		52		88		135		242	

Fatigue As Related to Total Performance

The purpose of this manner of handling the materials of the experiment is to see whether the work curve would be the same for those who accomplish much and for those who accomplish little in the total time; or whether the better workers fatigue more or less rapidly than the poorer workers.

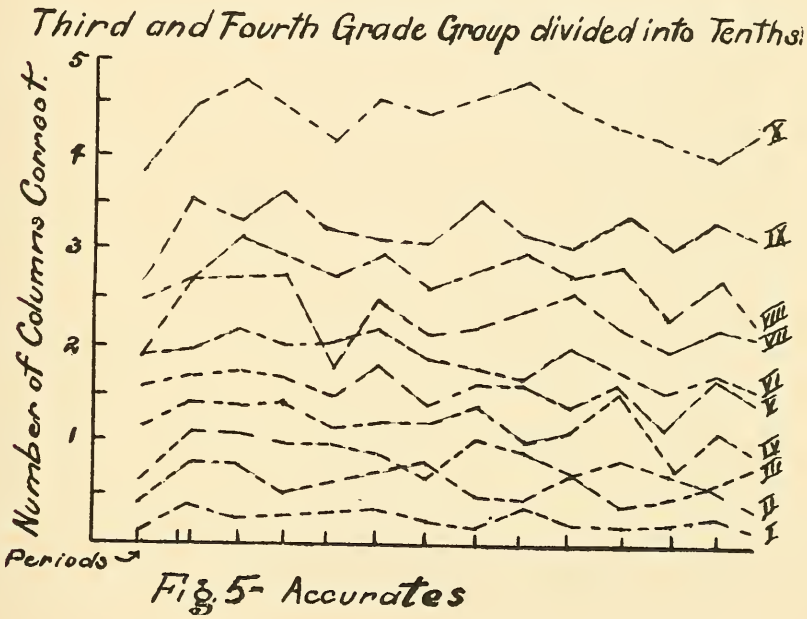


Fig. 5- Accurates

The two groups are divided into sub-groups on the basis of the total amount done in twenty-eight and forty-two minutes respectively. The 368 children of the Third and Fourth grades were arranged in rank order from lowest to highest, according to the total attempted in the twenty-eight minutes, and again according to the total accurate in the twenty-eight minutes, and this rank order arrangement was cut into ten parts. Each tenth of the 368 makes a sub-group, the sub-groups running from lowest to highest or upper tenth. The average of each sub-group was determined and curves were formed for these ten sub-groups. There are twenty

curves for the Third and Fourth grades. Likewise, there are twenty curves for the Seventh and Eighth grades. The 343 children in this group were arranged in rank order from lowest to

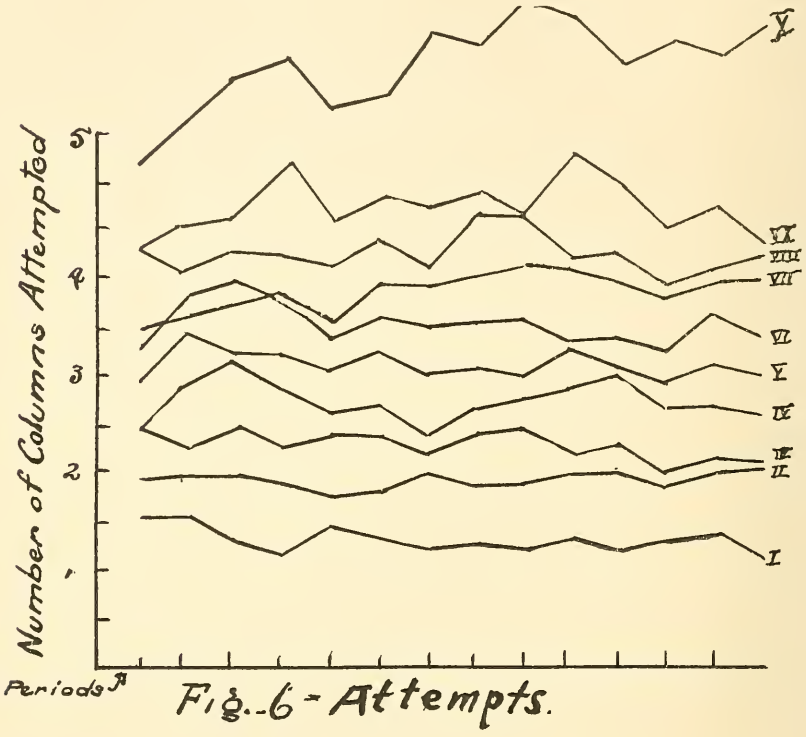


Fig. 6 - Attempts.

highest tenth or upper tenth, as in the case of the Third and Fourth grades and the numbers were treated in the same way.

These curves are handled in much the same way as the larger group curves. The idea here is to see if there is any difference in the form of a curve for a poor total accomplishment from that of the better accomplishment.

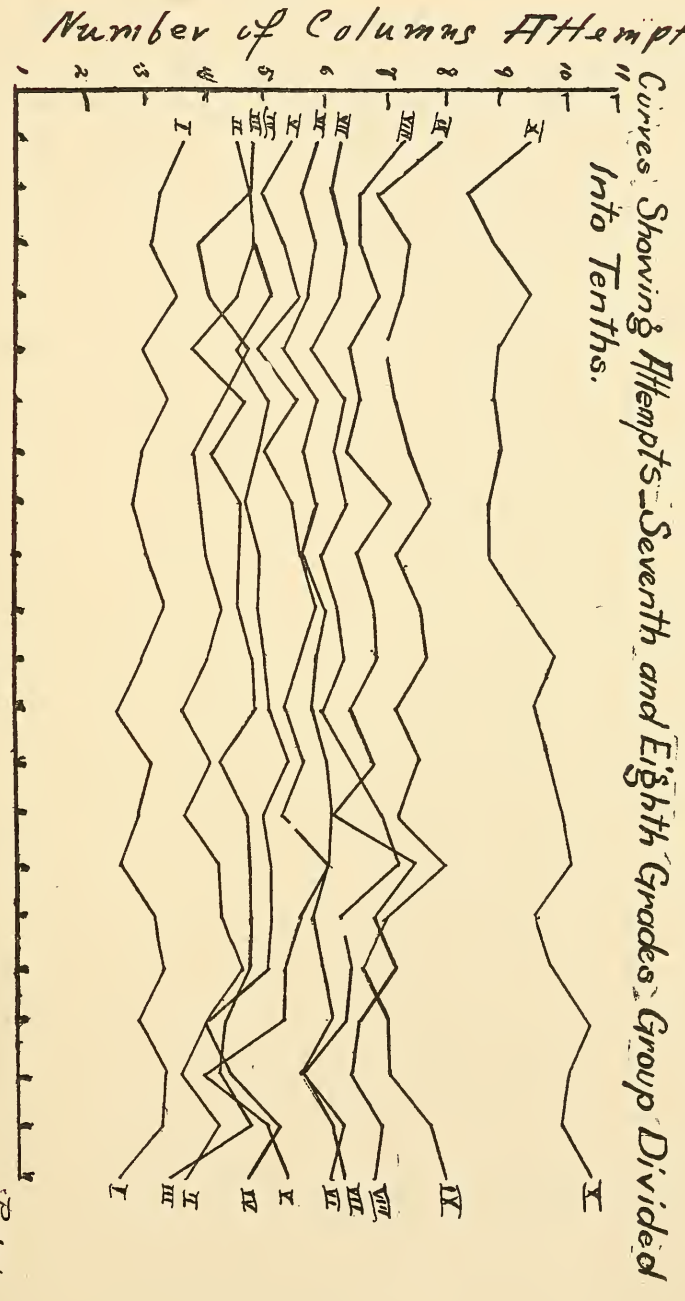

Curves Showing Attempts—Seventh and Eighth Grades Grouped Divided Into Tenths.
Number of Columns Attempted
Periods.
I
II
III
IV
V
VI
VII
VIII
IX
X

Fig. 7

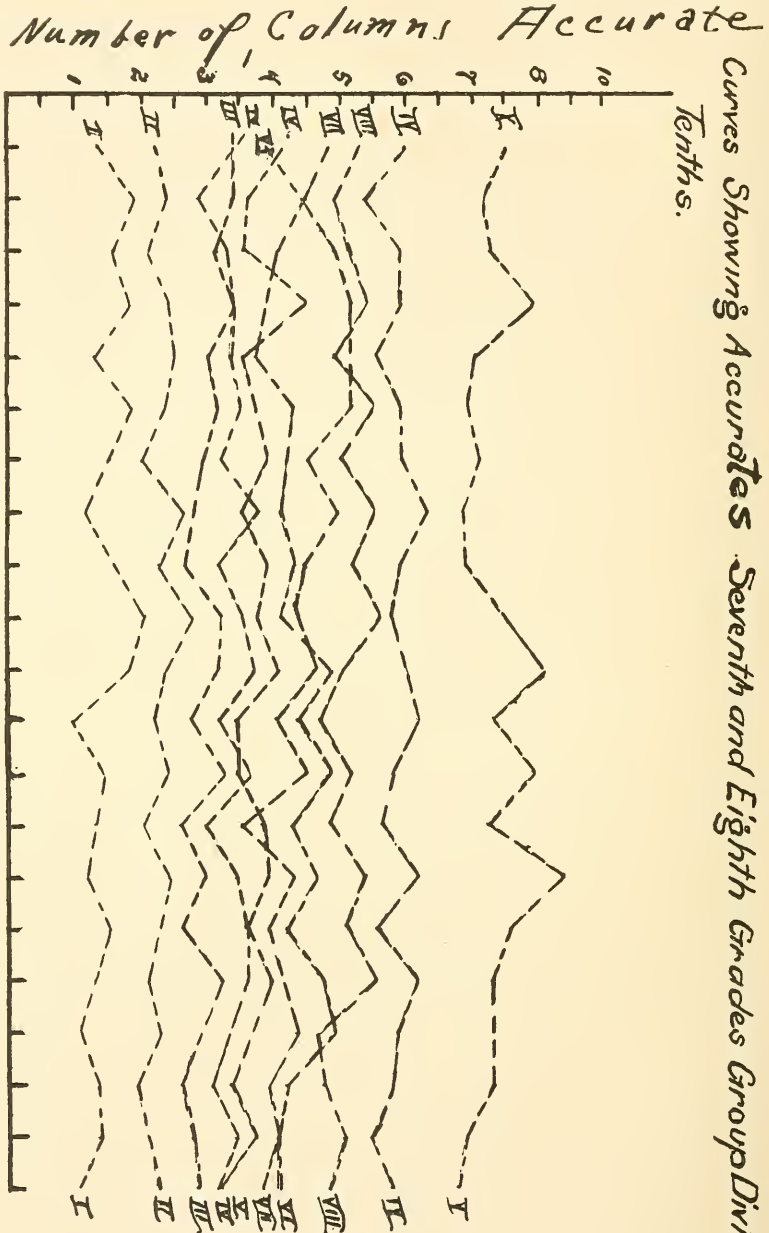


Fig. 8

Curves Showing Accuracies Seventh and Eighth Grades Grouped Divided Into Tenths.

TABLE XII

Attempts of Third and Fourth Grades

The 368 cases are divided into ten groups on the basis of total amount done in twenty-eight minutes, going from lowest tenth to upper tenth. Below are given the averages for each

First (Lowest) Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.52	.75	.11
2	1.5	.63	.09
3	1.27	.59	.08
4	1.19	.59	.08
5	1.44	.56	.08
6	1.36	.67	.09
7	1.22	.49	.07
8	1.33	.60	.08
9	1.22	.42	.06
10	1.36	.61	.09
11	1.22	.46	.06
12	1.33	.51	.07
13	1.38	.59	.07
14	1.08	.39	.05

Second Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.84	.55	.08
2	1.98	.41	.06
3	1.98	.31	.03
4	1.87	.41	.06
5	1.76	.51	.08
6	1.82	.46	.06
7	2.0	.27	.1
8	1.9	.42	.06
9	1.92	.5	.07
10	2.05	.45	.06
11	2.05	.3	.04
12	1.85	.46	.06
13	2.02	.36	.05
14	2.08	.6	.08

Third Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.43	.71	.1
2	2.29	.64	.09
3	2.49	.58	.08
4	2.37	.54	.08
5	2.4	.6	.09
6	2.52	.49	.07
7	2.27	.54	.07
8	2.4	.56	.08
9	2.45	.58	.08
10	2.29	.53	.07
11	2.37	.55	.07
12	2.02	.47	.07
13	2.21	.29	.04
14	2.08	.6	.08

Fourth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.43	.66	.09
2	2.92	.55	.08
3	3.16	.47	.07
4	2.84	.6	.08
5	2.63	.61	.09
6	2.79	.54	.08
7	2.43	.6	.08
8	2.76	.47	.07
9	2.79	.56	.08
10	2.9	.47	.07
11	3.02	.52	.08
12	2.71	.79	.11
13	2.73	.54	.08
14	2.71	.59	.08

TABLE XII (*Continued*)

Attempts of Third and Fourth Grades

Fifth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.95	.88	.12
2	3.41	.5	.07
3	3.29	.45	.06
4	3.24	.51	.07
5	3.05	.46	.06
6	3.35	.59	.08
7	3.	.37	.05
8	3.1	.52	.07
9	3.05	.46	.06
10	3.32	.58	.08
11	3.13	.51	.07
12	2.92	.5	.07
13	3.21	.54	.08
14	3.05	.75	.10

Sixth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.43	.79	.11
2	3.64	.68	.10
3	3.7	.58	.08
4	3.79	.50	.07
5	3.43	.59	.08
6	3.62	.52	.07
7	3.51	.58	.08
8	3.57	.6	.08
9	3.64	.56	.08
10	3.37	.65	.09
11	3.45	.68	.10
12	3.27	.55	.08
13	3.76	.61	.09
14	3.32	.76	.10

Seventh Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.37	.76	.11
2	3.86	.61	.09
3	3.92	.40	.06
4	3.86	.53	.07
5	3.57	.62	.09
6	3.98	.41	.06
7	3.92	.62	.09
8	4.05	.46	.06
9	4.18	.5	.07
10	4.16	.52	.07
11	4.05	.46	.06
12	3.87	.55	.08
13	4.02	.47	.07
14	4.05	.46	.06

Eighth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.29	.85	.12
2	4.02	.69	.10
3	4.34	.64	.09
4	4.29	.64	.09
5	4.16	.56	.08
6	4.40	.63	.09
7	4.13	.67	.10
8	4.56	.79	.11
9	4.64	.75	.10
10	4.29	.79	.11
11	4.32	.61	.09
12	3.94	.67	.093
13	4.16	.64	.089
14	4.34	.93	.132

TABLE XII (*Continued*)

Attempts of Third and Fourth Grades

Ninth Tenth—36 Cases				*Tenth Tenth—36 Cases*			
Period	*Average*	*A. D.*	*P. E.*	*Period*	*Average*	*A. D.*	*P. E.*
1	4.21	.85	.12	1	5.16	1.18	.17
2	4.61	.71	.10	2	5.75	1.11	.16
3	4.58	.79	.11	3	6.05	1.13	.16
4	5.22	.65	.09	4	6.25	1.13	.16
5	4.62	.61	.09	5	5.78	1.22	.17
6	4.89	.50	.07	6	5.39	1.09	.15
7	4.81	.65	.09	7	6.61	1.57	.22
8	4.95	.58	.08	8	6.58	1.47	.20
9	4.78	.72	.10	9	6.92	1.60	.22
10	5.36	.7	.10	10	6.75	1.56	.22
11	5.00	.55	.08	11	6.36	.94	.13
12	4.64	.73	.10	12	6.52	1.42	.20
13	4.78	.67	.09	13	6.41	1.48	.20
14	4.44	.78	.11	14	6.70	1.44	.20

TABLE XIII

Accurates of Third and Fourth Grades

The 368 cases are divided into ten groups on the basis of total amount done in twenty-eight minutes, going from lowest tenth to upper tenth. Below are given the averages for each group

First (Lowest) Tenth—37 Cases				*Second Tenth—37 Cases*			
Period	*Average*	*A. D.*	*P. E.*	*Period*	*Average*	*A. D.*	*P. E.*
1	.13	.26	.04	1	.37	.56	.08
2	.36	.49	.06	2	.81	.77	.11
3	.27	.40	.06	3	.81	.65	.09
4	.24	.36	.05	4	.51	.58	.08
5	.29	.43	.06	5	.67	.58	.08
6	.32	.43	.06	6	.70	.57	.08
7	.21	.34	.05	7	.78	.59	.08
8	.16	.27	.04	8	.59	.57	.08
9	.40	.56	.08	9	.48	.52	.08
10	.21	.34	.05	10	.72	.67	.09
11	.21	.34	.05	11	.40	.54	.08
12	.21	.33	.05	12	.59	.67	.09
13	.24	.52	.08	13	.62	.60	.08
14	.18	.31	.04	14	.43	.58	.08

TABLE XIII (*Continued*)

Accurates of Third and Fourth Grades

Third Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	.59	.64	.08
2	1.18	.81	.11
3	1.13	.58	.08
4	1.00	.7	.1
5	1.05	1.3	.18
6	.91	.59	.08
7	.75	.65	.08
8	1.08	1.00	.14
9	.97	.57	.08
10	.78	.72	.10
11	.82	.60	.08
12	.89	.74	.10
13	.64	.73	.10
14	.86	.56	.08

Fourth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.26	.82	.11
2	1.40	.83	.11
3	1.37	1.1	.15
4	1.43	.66	.09
5	1.13	.73	.10
6	1.24	.65	.09
7	1.24	.71	.10
8	1.43	.74	.10
9	1.05	.56	.08
10	1.18	.78	.11
11	1.55	.76	.11
12	.72	.76	.11
13	1.21	.79	.11
14	.90	.67	.09

Fifth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.55	.74	.10
2	1.71	.78	.10
3	1.79	.78	.10
4	1.79	.77	.10
5	1.55	.74	.10
6	1.87	.76	.10
7	1.43	.66	.09
8	1.60	.74	.10
9	1.60	.90	.13
10	1.45	.79	.10
11	1.65	.68	.09
12	1.18	.63	.09
13	1.68	.61	.09
14	1.49	.85	.13

Sixth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.90	1.04	.15
2	1.95	.88	.12
3	2.18	.91	.12
4	2.00	.70	.10
5	2.08	.64	.09
6	2.29	.80	.11
7	1.95	.77	.11
8	1.87	1.01	.14
9	1.68	.80	.11
10	2.02	.84	.11
11	1.73	.98	.14
12	1.63	.71	.10
13	1.73	.75	.11
14	1.63	1.03	.14

TABLE XIII (*Continued*)

Accurates of Third and Fourth Grades

Seventh Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.89	.88	.13
2	2.71	1.00	.14
3	2.73	1.03	.14
4	2.73	.72	.10
5	1.79	.61	.09
6	2.52	.84	.11
7	2.10	.62	.09
8	2.21	.83	.11
9	2.44	.88	.13
10	2.60	.79	.10
11	2.27	.92	.12
12	2.08	.91	.12
13	2.35	.84	.11
14	2.16	.80	.11

Eighth Tenth—37 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.49	1.04	.15
2	2.79	1.02	.14
3	3.16	.74	.10
4	2.98	.89	.12
5	2.79	.86	.12
6	3.00	.64	.09
7	2.68	.88	.12
8	2.82	1.01	.14
9	3.02	.79	.10
10	2.84	1.06	.14
11	2.95	.72	.10
12	2.29	.86	.12
13	2.79	.97	.14
14	2.29	.74	.10

Ninth Tenth—36 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.73	1.04	.15
2	3.57	.85	.12
3	3.33	.88	.12
4	3.63	.82	.12
5	3.27	.82	.12
6	3.10	.73	.10
7	3.13	.74	.10
8	4.08	1.36	.19
9	3.27	1.05	.15
10	3.16	.86	.12
11	3.45	.91	.12
12	3.08	.82	.12
13	3.40	.99	.14
14	3.27	1.01	.15

Tenth Tenth—36 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.84	.96	.13
2	4.5	1.16	.16
3	4.78	1.02	.15
4	4.41	1.19	.17
5	4.19	.93	.12
6	4.67	1.03	.15
7	4.47	1.02	.15
8	4.73	.82	.12
9	4.89	.96	.14
10	4.55	1.05	.15
11	4.33	1.14	.16
12	4.25	1.34	.19
13	4.08	1.36	.19
14	4.30	1.29	.18

TABLE XIV

Attempts of Seventh and Eighth Grades

The 343 cases are divided into ten groups on the basis of total amount done in forty-two minutes, going from lowest tenth to upper tenth. Below are given the averages for each group

First (Lowest) Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.71	1.13	.16
2	3.42	.89	.13
3	3.37	.65	.09
4	3.66	.87	.12
5	3.17	.68	.10
6	3.55	.73	.11
7	3.17	.61	.09
8	3.08	.63	.09
9	3.20	.56	.08
10	3.40	.57	.08
11	3.15	.57	.08
12	2.83	.64	.09
13	3.22	.55	.08
14	3.09	.68	.10
15	2.87	.64	.09
16	3.25	.70	.10
17	3.31	.77	.11
18	3.06	.70	.10
19	3.57	.98	.14
20	3.57	.84	.12
21	2.86	.65	.09

Second Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.65	1.24	.19
2	4.85	.89	.13
3	4.00	.74	.10
4	4.14	.63	.09
5	4.88	1.02	.14
6	4.20	.60	.09
7	3.95	.50	.07
8	4.00	.80	.12
9	4.11	.61	.09
10	4.33	.58	.08
11	4.22	.73	.10
12	3.83	.48	.07
13	4.17	.80	.12
14	3.8	.67	.10
15	4.34	.84	.12
16	4.31	.83	.11
17	4.58	.72	.10
18	4.11	.82	.12
19	3.75	.67	.10
20	4.31	.85	.12
21	4.85	1.05	.15

Third Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.88	.76	.09
2	4.74	.62	.12
3	4.83	.68	.11
4	4.68	.57	.12
5	3.88	.53	.11
6	4.68	.57	.07
7	4.21	.69	.11
8	4.77	.68	.09
9	4.71	.71	.07
10	4.98	.57	.08
11	4.79	.71	.10
12	4.38	.56	.08
13	5.00	.52	.10

Fourth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.89	1.03	.14
2	4.86	.51	.07
3	4.94	.66	.19
4	5.21	.57	.08
5	4.74	.63	.09
6	5.11	.73	.10
7	5.02	.51	.07
8	4.94	.67	.10
9	5.00	.70	.10
10	5.06	.78	.11
11	5.06	.50	.07
12	5.16	.80	.11
13	5.52	1.09	.15

TABLE XIV (*Continued*)

Attempts of Seventh and Eighth Grades

Third Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
14	4.17	.63	.10
15	4.62	.75	.10
16	4.74	.49	.08
17	4.77	.76	.08
18	4.65	.86	.08
19	4.44	.72	.10
20	4.83	.86	.09
21	3.59	.61	.11

Fourth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
14	5.17	.80	.12
15	5.26	.85	.12
16	5.29	.86	.12
17	5.21	.67	.19
18	4.12	1.04	.15
19	4.53	.99	.14
20	5.23	1.11	.16
21	4.86	.69	.10

Fifth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	5.53	1.34	.19
2	5.09	.86	.12
3	5.41	.69	.10
4	5.62	.75	.11
5	4.97	.86	.12
6	5.59	.67	.10
7	5.05	.72	.10
8	5.53	.73	.11
9	5.68	.62	.09
10	5.89	.86	.12
11	5.68	.83	.12
12	5.45	.64	.09
13	5.71	.75	.11
14	5.38	.62	.09
15	6.29	1.18	.17
16	5.74	.84	.12
17	5.71	.70	.10
18	5.45	.72	.10
19	4.42	1.06	.15
20	5.17	1.04	.15
21	5.38	.81	.12

Sixth Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	5.98	1.29	.19
2	5.75	.62	.09
3	5.89	.73	.11
4	5.80	.58	.08
5	5.52	.75	.11
6	5.92	.63	.09
7	5.72	.94	.14
8	5.92	.84	.12
9	5.63	.56	.08
10	6.08	.85	.12
11	5.92	.48	.07
12	5.89	.56	.08
13	6.02	.49	.07
14	6.20	.68	.10
15	6.22	1.06	.15
16	5.78	.68	.10
17	5.92	.74	.11
18	6.17	.83	.12
19	5.80	.58	.08
20	6.36	.90	.13
21	6.22	.74	.11

Seventh Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	6.26	1.00	.14
2	6.17	.75	.11
3	6.32	.92	.13
4	6.26	.70	.10
5	5.74	.67	.10

Eighth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	7.29	.98	.13
2	6.68	.92	.13
3	6.77	.59	.08
4	6.95	.72	.10
5	6.41	.62	.09

TABLE XIV (*Continued*)

Attempts of Seventh and Eighth Grades

Seventh Tenth—34 Cases				*Eighth Tenth—34 Cases*			
Period	*Average*	*A. D.*	*P. E.*	*Period*	*Average*	*A. D.*	*P. E.*
6	6.38	.86	.12	6	6.62	.62	.09
7	6.33	.74	.11	7	6.45	.65	.09
8	6.35	.84	.12	8	7.05	.66	.10
9	5.92	.52	.07	9	6.71	.59	.08
10	6.23	.81	.12	10	6.80	.61	.09
11	6.32	.77	.11	11	6.83	.61	.09
12	6.00	.76	.11	12	6.59	.64	.09
13	6.48	.88	.13	13	6.74	.62	.09
14	6.98	.97	.14	14	6.38	.99	.14
15	7.23	1.14	.16	15	7.55	1.00	.14
16	6.44	.87	.13	16	6.95	.78	.11
17	6.58	.93	.13	17	7.21	.89	.13
18	6.41	.72	.10	18	6.69	.78	.11
19	5.68	.86	.12	19	6.62	.70	.10
20	6.26	1.19	.17	20	7.05	.78	.11
21	6.35	1.02	.15	21	6.92	.98	.14

Ninth Tenth—34 Cases				*Tenth Tenth—34 Cases*			
Period	*Average*	*A. D.*	*P. E.*	*Period*	*Average*	*A. D.*	*P. E.*
1	7.83	1.13	.16	1	9.29	1.26	.18
2	6.83	.93	.13	2	8.61	1.15	.17
3	7.59	.78	.11	3	8.86	1.18	.17
4	7.50	.79	.11	4	9.44	1.31	.19
5	7.14	.71	.10	5	8.92	1.39	.20
6	7.29	.59	.08	6	8.89	1.09	.16
7	7.41	.83	.12	7	8.98	1.15	.17
8	7.76	.92	.13	8	8.95	1.12	.16
9	7.26	.69	.10	9	8.83	1.04	.15
10	7.65	.82	.12	10	9.38	1.25	.18
11	7.74	.78	.11	11	9.27	1.30	.19
12	7.25	.78	.11	12	9.52	1.61	.23
13	7.62	.79	.11	13	9.77	1.60	.23
14	7.45	.88	.13	14	9.95	1.41	.20
15	8.27	1.48	.21	15	10.11	1.60	.23
16	7.02	.62	.09	16	9.68	1.73	.25
17	6.77	.79	.11	17	9.98	1.50	.22
18	7.28	1.05	.15	18	10.44	2.18	.31
19	7.29	1.01	.14	19	10.11	1.56	.22
20	7.92	.65	.09	20	10.14	2.09	.30
21	8.14	1.25	.18	21	10.79	2.59	.37

TABLE XV

Accurates of Seventh and Eighth Grades

The 343 cases are divided into ten groups on the basis of total amount done in forty-two minutes, going from lowest tenth to upper tenth. Below are given the averages for each group

First (Lowest) Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	1.43	1.42	.07
2	1.86	1.37	.06
3	1.77	1.19	.05
4	1.80	.96	.04
5	1.49	1.01	.05
6	1.89	.98	.04
7	1.67	1.03	.06
8	1.52	.92	.04
9	1.80	.88	.04
10	2.17	.85	.04
11	1.83	.83	.04
12	1.02	.66	.03
13	1.54	1.04	.06
14	1.45	.97	.04
15	1.40	.97	.04
16	1.58	.82	.04
17	1.48	1.04	.06
18	1.22	.84	04
19	1.37	.95	.04
20	1.31	.93	.04
21	1.14	.71	.03

Second Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	2.22	1.00	.14
2	2.37	.98	.14
3	2.20	1.09	.16
4	2.40	1.00	.14
5	2.55	.62	.09
6	2.49	.52	.07
7	2.08	.96	.13
8	2.69	1.23	.17
9	2.40	.85	.12
10	2.75	.73	.10
11	2.43	.93	.13
12	2.20	.77	.11
13	2.45	.94	.13
14	2.00	.85	.12
15	2.45	.92	.13
16	2.31	1.15	.16
17	2.28	1.04	.15
18	2.34	1.02	.15
19	2.02	.83	.12
20	2.05	.81	.12
21	2.20	1.08	.16

Third Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.32	1.06	.18
2	2.95	.88	.12
3	3.26	.80	.12
4	3.41	.82	.12
5	3.08	.99	.14
6	3.05	1.06	.15
7	3.00	.94	.13
8	2.89	1.09	.16
9	2.74	.91	.13
10	3.17	1.05	.16
11	3.17	1.09	.16
12	2.54	1.13	.16
13	3.14	.88	.13

Fourth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.35	1.10	.16
2	3.36	.78	.11
3	3.17	1.10	.16
4	3.48	.82	.12
5	3.44	.79	.10
6	3.50	.82	.12
7	3.23	.84	.12
8	3.71	.92	.13
9	3.23	.87	.12
10	3.53	1.05	.15
11	3.68	.94	.13
12	3.17	.95	.14
13	3.75	1.27	.18

TABLE XV (*Continued*)

Accurates of Seventh and Eighth Grades

Third Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
14	2.69	1.08	.16
15	2.74	.89	.13
16	2.66	1.05	.15
17	3.08	1.06	.15
18	2.83	.84	.13
19	2.44	1.02	.15
20	2.71	.93	.13
21	2.77	1.02	.15

Fourth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
14	2.95	.95	.14
15	3.38	1.01	.15
16	3.53	.99	.14
17	3.59	.99	.14
18	3.33	1.09	.15
19	3.11	.76	.11
20	3.14	1.30	.19
21	3.11	1.18	.16

Fifth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.09	1.10	.16
2	3.68	1.20	.15
3	3.56	.93	.13
4	4.58	.78	.11
5	3.59	.96	.13
6	3.68	.98	.14
7	3.77	.90	.13
8	3.65	.90	.13
9	3.91	.78	.11
10	3.80	.94	.13
11	4.00	1.11	.16
12	3.40	1.08	.15
13	3.50	.97	.14
14	3.80	1.19	.17
15	3.86	1.06	.15
16	3.62	.97	.14
17	3.80	.80	.12
18	3.65	1.23	.18
19	3.11	.96	.14
20	3.59	1.41	.20
21	3.20	1.41	.20

Sixth Tenth—35 Cases

Period	*Average*	*A. D.*	*P. E.*
1	3.98	1.49	.21
2	4.44	1.11	.16
3	4.17	1.03	.15
4	3.89	1.01	.15
5	3.67	1.30	.19
6	4.11	1.14	.16
7	4.26	1.20	.17
8	4.80	1.12	.16
9	4.36	.94	.14
10	4.11	1.20	.17
11	4.62	.95	.14
12	4.00	.68	.10
13	4.50	1.18	.16
14	3.68	1.10	.16
15	4.23	1.21	.17
16	4.00	1.14	.16
17	4.14	.93	.13
18	4.26	1.22	.18
19	4.11	.77	.11
20	4.13	1.01	.15
21	4.12	1.30	.19

Seventh Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	4.66	1.55	.22
2	4.58	1.29	.19
3	4.89	1.17	.17
4	5.09	.96	.13
5	5.06	.84	.12

Eighth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	5.29	1.12	.17
2	4.92	.76	.11
3	5.05	1.12	.16
4	5.44	1.05	.15
5	4.83	1.19	.17

TABLE XV (*Continued*)

Accurates of Seventh and Eighth Grades

Seventh Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
6	5.03	.57	.08
7	4.41	1.04	.15
8	4.77	.63	.09
9	4.48	1.00	.14
10	4.38	1.25	.17
11	4.76	1.31	.19
12	4.23	1.02	.15
13	4.83	1.07	.15
14	4.29	1.24	.17
15	4.74	1.20	.17
16	4.29	1.08	.16
17	4.74	1.02	.15
18	4.86	1.04	.15
19	4.26	1.28	.17
20	4.11	1.08	.16
21	4.00	1.29	.18

Eighth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
6	5.50	1.08	.16
7	5.17	.93	.13
8	5.58	1.39	.19
9	5.08	1.06	.15
10	5.59	1.08	.16
11	5.00	.94	.13
12	4.71	1.12	.16
13	5.20	.90	.13
14	4.95	1.12	.16
15	5.35	1.41	.16
16	5.17	1.29	.18
17	5.58	1.03	.15
18	4.65	1.13	.16
19	4.77	.92	.13
20	5.11	1.18	.17
21	5.00	1.11	.16

Ninth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	5.95	1.37	.19
2	5.45	1.09	.16
3	5.95	1.00	.14
4	5.92	.77	.11
5	5.59	1.01	.15
6	5.83	.98	.13
7	5.86	.82	.12
8	6.29	1.01	.15
9	5.92	.87	.13
10	5.89	1.06	.15
11	5.95	1.12	.16
12	6.08	.99	.13
13	5.83	.98	.13
14	5.80	1.11	.16
15	5.24	1.22	.17
16	5.56	1.16	.17
17	6.23	.87	.13
18	5.98	1.09	.16
19	5.83	.95	.13
20	5.55	1.08	.16
21	5.77	1.07	.15

Tenth Tenth—34 Cases

Period	*Average*	*A. D.*	*P. E.*
1	7.59	1.43	.20
2	7.13	1.25	.17
3	7.23	1.51	.16
4	7.80	1.36	.20
5	7.14	1.08	.16
6	6.89	1.49	.20
7	7.08	1.03	.15
8	6.92	1.27	.17
9	6.98	1.15	.16
10	7.42	1.45	.20
11	8.02	1.33	.20
12	7.35	1.35	.20
13	7.95	1.54	.16
14	7.32	1.43	.20
15	8.32	1.66	.24
16	7.62	1.29	.17
17	7.38	1.74	.25
18	7.32	1.83	.26
19	7.17	1.40	.20
20	7.17	1.20	.16
21	6.83	1.53	.16

TABLE XVI

Third and Fourth Grades.—Attempts

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
1	1	14	1.43	1.26	12% loss	1.37	1.31	1.27	1.00
									.95
									.92
2	14	5	1.93	1.99	3% gain	1.92	1.88	2.01	1.00
									.97
									1.04
3	6	12	2.40	2.10	12% loss	2.40	2.41	2.19	1.00
									1.00
									.91
4	3	1 and 7	2.84	2.72	4% loss	2.84	2.68	2.81	1.00
									.94
									.99
5	2	12	3.22	3.06	5% loss	3.22	3.11	3.13	1.00
									.96
									.97
6	4	12	3.59	3.45	4% loss	3.64	3.55	3.43	1.00
									.96
									.94
7	9	1	3.72	3.98	7% gain	3.75	3.94	4.03	1.00
									1.05
									1.07
8	8	12	4.22	4.16	1% loss	4.24	4.40	4.22	1.00
									1.037
									.99
9	10	1	4.47	4.62	3% gain	4.65	4.81	4.84	1.00
									1.034
									1.04
10	9	1	5.65	6.54	16% gain	5.80	6.36	6.55	1.00
									1.09
									1.13

Third and Fourth Grades—Accurates

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
1	9	1	.25	.21	16% loss	.25	.28	.21	1.00
									1.12
									.84
2	2	1	.66	.55	17% loss	.63	.64	.55	1.00
	3								1.00
									.87

TABLE XVI (*Continued*)

Third and Fourth Grades—Accurates

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
3	2	1	.97	.80	18% loss	.98	.95	.80	1.00 .96 .81
4	11	12	1.34	.94	30% loss	1.36	1.22	1.11	1.00 .89 .81
5	6	12	1.68	1.45	14% loss	1.71	1.61	1.49	1.00 .94 .87
6	6	14	2.01	1.66	17% loss	2.01	1.97	1.75	1.00 .98 .87
7	3 4	5	2.44	2.20	10% loss	2.51	2.21	2.29	1.00 .88 .91
8	3	12	2.81	2.46	12% loss	2.85	2.86	2.63	1.00 1.003 .92
9	8	1	3.21	3.25	1% gain	3.32	3.37	3.27	1.00 1.01 .98
10	9	1	4.37	4.21	4% loss	4.38	4.59	4.30	1.00 1.04 .98

TABLE XVII

Seventh and Eighth Grades—Attempts

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
1	1	12	3.50	3.53	5% loss	3.44	3.14	3.21	1.00 .91 .93
2	5	19	4.50	4.30	5% loss	4.38	4.07	4.32	1.00 .93 .98
3	13	21	4.82	4.29	11% loss	4.70	4.69	4.66	1.00 .99 .99
4	13	18	4.90	4.87	1% loss	4.97	5.13	4.93	1.00 1.03 .99

TABLE XVII (*Continued*)

Seventh and Eighth Grades—Attempts

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
5	15	19	5.34	4.99	7% loss	5.32	5.62	5.45	1.00
									1.05
									1.02
6	20	5	5.87	6.13	4% gain	5.80	5.95	6.07	1.00
									1.02
									1.04
7	15	19	6.25	6.10	2% loss	6.21	6.23	6.42	1.00
									1.00
									1.03
8	15	14	6.91	6.86	1% loss	6.74	6.73	7.01	1.00
									.99
									1.04
9	15	17	7.42	7.78	5% gain	7.37	7.53	7.53	1.00
									1.02
									1.02
10	21	2	8.92	10.35	16% gain	9.10	9.38	10.18	1.04
									1.03
									1.12

Seventh and Eighth Grades—Accurates

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
1	10	12	1.69	1.27	25% loss	1.70	1.62	1.36	1.00
									.95
									.80
2	10	14	2.26	2.09	8% loss	2.33	2.42	2.23	1.00
									1.08
									.95
3	4	19	3.18	2.64	17% loss	3.15	2.91	2.75	1.00
									.92
									.87
4	13	14	3.29	3.12	5% loss	3.36	3.43	3.31	1.00
									1.02
									.98
5	4	19	3.78	3.30	13% loss	3.85	3.23	3.55	1.00
									.84
									.92
6	8	5	4.20	4.12	2% loss	4.07	4.29	4.14	1.00
									1.05
									1.01
7	4	21	4.70	4.12	12% loss	4.82	4.63	4.42	1.00
									.93
									.91

TABLE XVII (*Continued*)

Seventh and Eighth Grades—Accurates

Sub-Groups	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*
8	10	18	5.09	4.96	3% loss	5.17	5.16	5.09	1.00
									.99
									.98
9	8	2	5.78	5.72	1% loss	5.79	5.97	5.88	1.00
									1.03
									1.01
10	15	21	7.32	7.11	3% loss	7.27	7.42	7.42	1.00
									1.02
									1.02

EXPLANATION OF TABLES NOS. XVI AND XVII

In these tables we have under

I. The maximum period for each curve.
II. The minimum period for each curve.
III. Average columns for first three periods.
IV. Average columns for last three periods.
V. Per cent. of gain or loss when III and IV are compared with III as a basis.
VI. Average of first third of curve.
VII. Average of second third of curve.
VIII. Average of last third of curve.
IX. Relative value of VI, VII, and VIII, using VI as a basis.

The figures under V show that the group fatigued or did not fatigue—according as there was loss or gain. IX gives an idea of the general form of the curve.

To begin with the facts as summarized in Table XVI. The location of the maximum (Column I) is certainly later with the better workers, when attempts are considered, in either the Third and Fourth or the Seventh and Eighth grades; the minimum, on the other hand, tends to come earlier with the better groups than with the poorer. In the case of accurates, no clear relation appears between the location of the maximum or minimum and the total accomplishment.

Column V shows pretty clearly that the losses tend to be smaller towards the bottom of the column, which means that the better workers fatigued less than the poorer workers. In the case of attempts, the loss actually gives way to a gain, while in the case of accurates, the best workers lose very little at the end as compared with the beginning. Column IX, attentively considered, will show the same thing.

But, undoubtedly, the best presentation of the results is afforded by the curves.[1] The curves for attempts, in both the younger and the older group, show a clear general rise in the case of the upper tenth, indicating that the best workers increased their speed during the experiment; while the curve for the lowest tenth shows the

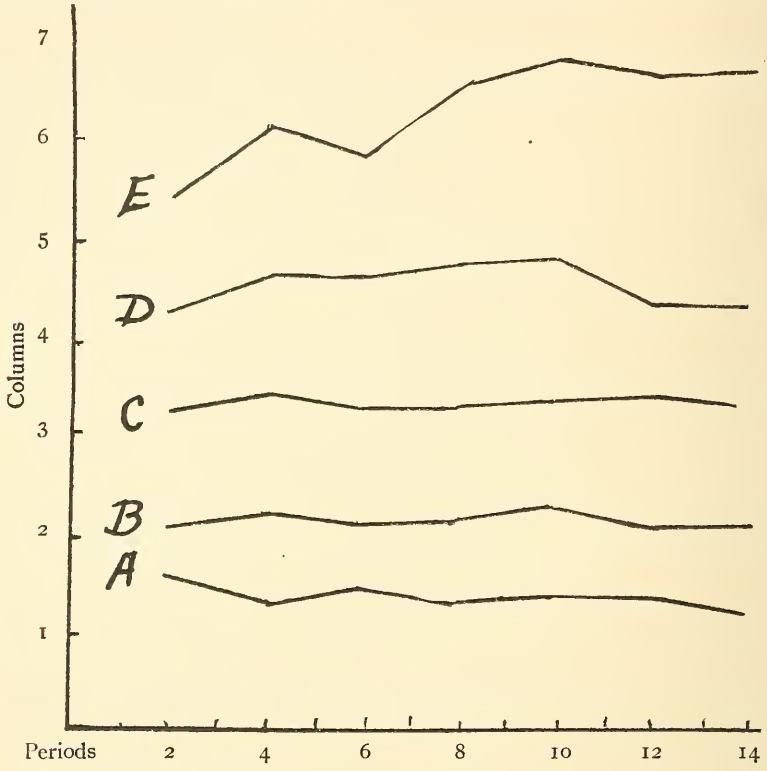


Fig. 9. Attempts—Third and Fourth Grades

EXPLANATION OF FIGURES 9–12

The points on these curves are for the average performances for every four minutes, the data being handled thus so as to smooth out the curves.

opposite course. The curves for the intervening tenths are rather confusing, but apparently have no definite tendency to deviate much from the horizontal—which would indicate that, on the whole, the medium workers neither increased nor decreased their speed.

[1] See Figures 5, 6, 7, and 8 on pages 31, 32, 33 and 34

The curves for accurates fail to reveal to the eye any clear difference between the sub-groups.

Evidently we have made too many sub-groups in dividing into tenths. While highest and lowest tenths should evidently be kept separate, as their curves stand so definitely apart from the rest, the second and third tenths may very well be combined, and likewise the eighth and ninth, leaving the four middle tenths to com-

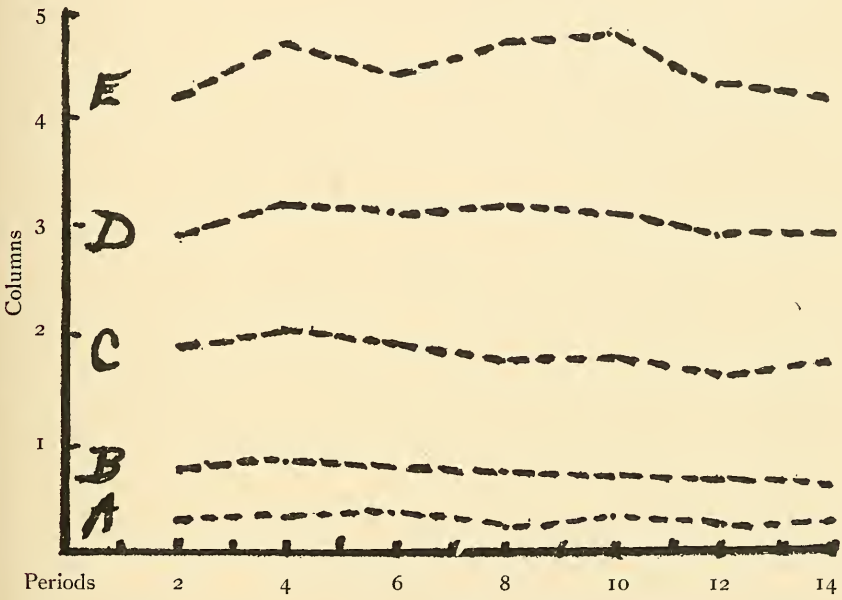


Fig. 10. Accurates—Third and Fourth Grades

bine into one central group. The resulting division into five unequal parts gives approximately equal spaces between the averages of the adjacent groups, as should be the case according to the theory of distribution, and as appears in fact to be the case in the following curves constructed as above indicated. In order further to eliminate irrelevant fluctuations, the points on the curves indicate the averages of two adjacent periods of two minutes. The Tables, XVIII to XXI, give the averages for each period. Group A, in these tables, consists of the lowest tenth, Group B of the second

and third tenths, Group C of the fourth to seventh tenths, Group D of the eighth and ninth, and Group E of the uppermost tenth.

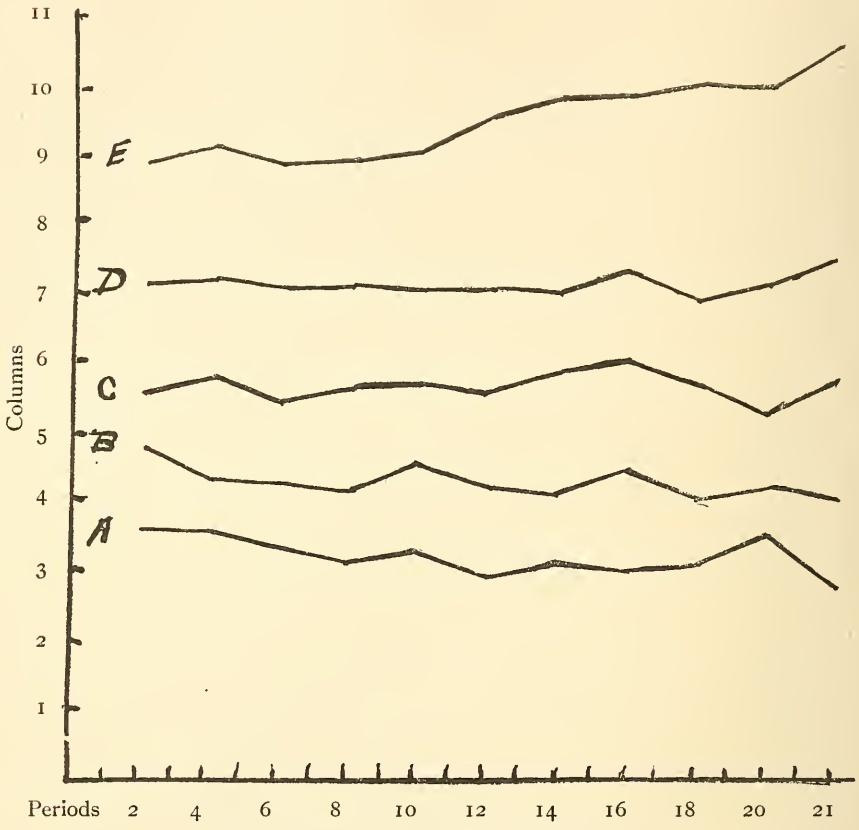


Fig. 11. Attempts—Seventh and Eighth Grades

The curves, thus reduced to their lowest terms, indicate pretty clearly that there is a relation in attempts between speed of work and liability to fatigue. The most rapid workers increase their speed as time goes by, while the slowest workers fall off, the middle groups remaining, on the average, almost perfectly steady for the time here considered. In accurate work, the general tendency of the whole group was shown in the last chapter to be slightly downward towards the end. But the decline comes later with the upper

group, and is somewhat less pronounced than in the lowermost group, at least when the decline is considered in relation to the height of the curve above the base line. This is brought out by another way of examining the results.

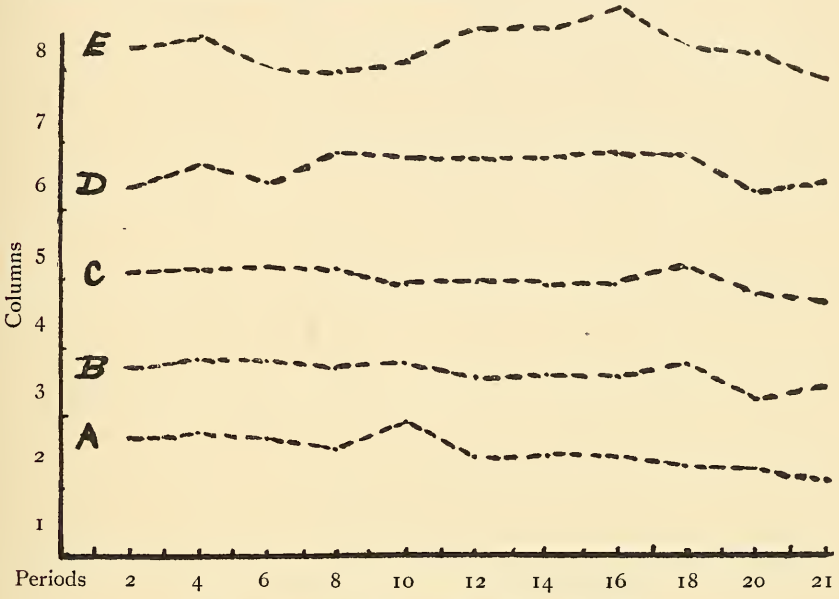


Fig. 12. Accurates—Seventh and Eighth Grades

As heretofore, the first three periods were compared with the last three periods. The results are given below:

Third and Fourth Grades

Attempts	*Accurates*
Group A, 12 per cent. loss	Group A, 16 per cent. loss
Group B, 5 per cent. loss	Group B, 18 per cent. loss
Group C, 1 per cent. loss	Group C, 18 per cent. loss
Group D, 1 per cent. gain	Group D, 6 per cent. loss
Group E, 16 per cent. gain	Group E, 4 per cent. loss

Seventh and Eighth Grades

Attempts	*Accurates*
Group A, 5 per cent. loss	Group A, 25 per cent. loss
Group B, 8 per cent. loss	Group B, 12 per cent. loss
Group C, 1 per cent. loss	Group C, 8 per cent. loss
Group D, 2 per cent. gain	Group D, 2 per cent. loss
Group E, 16 per cent. gain	Group E, 3 per cent. loss

In the case of attempts, the relation between total speed and fatigue comes out in this tabulation as clearly as before. The loss shown by the slowest groups gives way to a gain with the quickest workers. In the case of accurates, the change is certainly less clear, and there is no passing over to a gain; yet it may fairly be said that the better groups show a smaller loss than the poorer.

The facts of Column IX in Tables XVI and XVII are taken for the Groups A, B, C, D, and E of the previous tabulation. The facts for the lowest tenth are the facts for Group A, the second and third tenth are those for Group B, etc.

Third and Fourth Grades

Attempts Group	1	2	3	*Accurates* Group	1	2	3
A	1.00	.95	.92	A	1.00	1.12	.84
B	1.00	.985	.975	B	1.00	.98	.84
C	1.00	.99	.99	C	1.00	.92	.87
D	1.00	1.035	1.015	D	1.00	1.01	.95
E	1.00	1.09	1.13	E	1.00	1.04	.98

Seventh and Eighth Grades

Attempts Group	1	2	3	*Accurates* Group	1	2	3
A	1.00	.91	.93	A	1.00	.95	.80
B	1.00	.96	.985	B	1.00	1.00	.91
C	1.00	1.028	1.02	C	1.00	.96	.955
D	1.00	1.005	1.03	D	1.00	1.01	1.005
E	1.00	1.03	1.12	E	1.00	1.02	1.02

In the case of attempts, the relation between total speed and liability to fatigue is about as clear in this present tabulation as it was in the previous one. The quickest workers gain and the poorest workers lose. In the case of accuracy the change is clearer, for in the younger classes the better workers show a smaller loss than the poorer ones and in the older classes the best workers show a small gain. Consequently this way of examining the results supports the conclusion that on the whole the slower workers fatigue more quickly than the more rapid workers.

This, then, is put forward as one of the chief results of the experiment: that, on the whole, *the slower workers fatigue more quickly than the more rapid workers.*

TABLE XVIII

Attempts, Third and Fourth Grades

	Group I	*Groups II, III*	*Groups IV, V, VI, VII,*	*Groups VIII, IX*	*Group X*
	A	*B*	*C*	*D*	*E*
1	1.52	2.13	3.04	4.25	5.16
2	1.50	2.13	3.46	4.31	5.75
3	1.27	2.23	3.52	4.41	6.05
4	1.19	2.12	3.43	4.75	6.25
5	1.44	2.08	3.17	4.39	5.78
6	1.36	2.17	3.43	4.65	5.92
7	1.22	2.17	3.21	4.47	6.61
8	1.33	2.15	3.34	4.82	6.58
9	1.22	2.19	3.34	4.72	6.92
10	1.36	2.17	3.44	4.82	6.75
11	1.22	2.21	3.66	4.66	6.36
12	1.33	1.95	3.19	4.29	6.52
13	1.38	2.11	3.43	4.48	6.41
14	1.08	2.12	3.28	4.39	6.70

TABLE XIX

Accurates, Third and Fourth Grades

	Group I	*Groups II, III*	*Groups IV, V, VI, VII*	*Groups VIII, IX*	*Group X*
	A	*B*	*C*	*D*	*E*
1	.13	.51	1.65	2.61	3.84
2	.35	1.00	1.94	3.18	4.50
3	.27	.97	2.02	3.24	4.78
4	.24	.76	1.99	3.30	4.41
5	.29	.86	1.64	3.03	4.19
6	.32	.81	1.98	3.05	4.67
7	.21	.77	1.68	2.90	4.47
8	.16	.86	1.78	3.45	4.73
9	.40	.73	1.69	3.14	4.89
10	.21	.77	1.81	3.00	4.55
11	.21	.60	1.80	3.20	4.33
12	.21	.74	1.40	2.68	4.25
13	.24	.63	1.74	3.09	4.08
14	.18	.65	1.55	2.78	4.30

TABLE XX

Attempts—Seventh and Eighth Grades

	Group I	*Groups II, III*	*Groups IV, V, VI, VII*	*Groups VIII, IX*	*Group X*
	A	*B*	*C*	*D*	*E*
1	3.71	4.77	5.66	7.56	9.29
2	3.42	4.79	5.47	6.75	8.61
3	3.37	4.43	5.64	7.18	8.86
4	3.66	4.41	5.72	7.22	9.44
5	3.17	4.38	5.24	6.77	8.93
6	3.55	4.44	5.75	7.45	8.89
7	3.17	4.08	5.63	6.93	8.98
8	3.08	4.39	5.68	7.40	8.95
9	3.20	4.41	5.55	6.98	8.83
10	3.40	4.65	5.86	7.22	9.38
11	3.15	4.50	5.74	7.28	9.77
12	2.83	4.11	5.60	6.92	9.52
13	3.22	4.59	5.93	7.18	9.77
14	3.09	3.99	5.93	6.92	9.95
15	2.87	4.48	6.25	7.91	10.11
16	3.25	4.52	5.81	6.98	9.68
17	3.31	4.68	5.85	6.94	9.98
18	3.06	4.38	5.54	7.03	10.44
19	3.57	4.09	5.11	6.95	10.11
20	3.57	4.57	5.73	7.48	10.14
21	2.86	4.22	5.70	7.53	10.79

TABLE XXI

Accurates, Seventh and Eighth Grades

	Group I	*Groups II, III*	*Groups IV, V, VI, VII*	*Groups VIII, IX*	*Group X*
	A	*B*	*C*	*D*	*E*
1	1.43	2.72	4.02	5.62	7.59
2	1.86	2.66	4.01	5.19	7.13
3	1.77	2.73	3.89	5.50	7.23
4	1.80	2.90	4.26	5.68	7.80
5	1.49	2.82	4.19	5.21	7.14
6	1.89	2.77	4.08	5.66	6.89
7	1.67	2.54	3.92	5.56	7.08
8	1.52	2.79	4.24	5.94	6.92
9	1.80	2.57	3.99	5.50	6.98
10	2.17	2.96	3.95	5.74	7.42
11	1.83	2.80	4.26	5.47	8.02
12	1.02	2.37	3.70	5.39	7.35
13	1.54	2.79	4.14	5.52	7.95
14	1.45	2.34	3.68	5.37	7.32
15	1.40	2.59	4.05	5.79	8.32
16	1.58	2.48	3.85	5.36	7.62
17	1.48	2.68	4.07	5.90	7.38
18	1.22	2.58	4.02	5.32	7.32
19	1.37	2.23	3.65	5.30	7.32
20	1.31	2.38	3.77	5.33	7.17
21	1.14	2.49	3.58	5.39	6.83

CHAPTER IV

VARIABILITY OF THE INDIVIDUAL

Dependence of Variability upon Central Tendency

Everyone feels that variability is bound to increase as the measure of performance increases. There is evidently some dependence of variability upon the absolute measure of performance, such that the greater the absolute measure of performance the greater the variability.

Pearson has accepted the common-sense idea regarding this dependence, *i.e.*, that variability is normally or typically proportional to the absolute measure of performance. Fullerton and Cattell, and Thorndike, following the rule governing the variable error in the theory of measurement, have held that the variability normally increases as the square root of the absolute measure of performance, though probably with modifications in different kinds of performance.

Our data afford the opportunity of examining the variability of each individual from one period to another of the fourteen or twenty-one two-minute periods of the test, and then comparing the variability of individuals with smaller and greater total (or average) performance. This would be little to the purpose if a pronounced practice or fatigue effect occurred within the limits of the work; but, as these effects have been shown to be rather slight, the variability that appears in the individual's performance can be accepted as, in the main, simply the ordinary variability of performance.

TABLE XXII

Accurates of Third and Fourth Grades, showing average performance and variability of each individual for the twenty-eight minutes

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
0			
0			
1	.07	.13	
1	.07	.13	
1	.07	.13	
1	.07	.13	
1	.07	.13	.13
2	.14	.25	
2	.14	.25	
2	.14	.25	
2	.14	.25	
2	.14	.25	.25
3	.21	.34	
3	.21	.34	
3	.21	.34	
3	.21	.34	
3	.21	.34	.34
4	.29	.42	
4	.29	.42	
4	.29	.42	
4	.29	.46	
4	.29	.46	
4	.29	.46	.44
5	.36	.73	
5	.36	.73	
5	.36	.66	
5	.36	.66	
5	.36	.66	
5	.36	.66	
5	.36	.66	.68
6	.43	.63	
6	.43	.57	
6	.43	.57	
6	.43	.63	
6	.43	.57	
6	.43	.50	
6	.43	.63	
6	.43	.57	.58
7	.50	.64	
7	.50	.61	
7	.50	.61	

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
7	.50	.54	
7	.50	.54	
7	.50	.61	
7	.50	.61	
7	.50	.74	
7	.50	.54	.60
8	.57	.60	
8	.57	.52	
8	.57	.67	
8	.57	.60	
8	.57	.60	
8	.57	.60	.60
9	.64	.57	
9	.64	.73	
9	.64	.64	
9	.64	.42	
9	.64	.53	
9	.64	.53	
9	.64	.65	
9	.64	.53	.58
10	.71	.52	
10	.71	.46	
10	.71	.63	
10	.71	.75	
10	.71	.82	
10	.71	.75	
10	.71	.75	
10	.71	.68	
10	.71	.72	
10	.71	1.04	
10	.71	.72	
10	.71	.72	
10	.71	.81	.72
11	.79	.67	
11	.79	.69	
11	.79	.97	
11	.79	.81	
11	.79	.91	.81
12	.86	.48	
12	.86	.74	
12	.86	.34	
12	.86	.63	
12	.86	.74	
12	.86	.61	

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
12	.86	.52	
12	.86	.52	
12	.86	1.52	
12	.86	.73	
12	.86	.73	.69
13	.93	.80	
13	.93	.53	
13	.93	.68	
13	.93	.70	
13	.93	.80	
13	.93	.70	
13	.93	.57	
13	.93	.68	
13	.93	.52	
13	.93	1.05	
13	.93	.70	.70
14	1.00	.44	
14	1.00	.73	
14	1.00	.29	
14	1.00	.73	
14	1.00	.58	
14	1.00	.87	
14	1.00	.44	
14	1.00	.73	
14	1.00	.86	
14	1.00	.58	
14	1.00	.57	
14	1.00	.71	.63
15	1.07	.92	
15	1.07	.63	
15	1.07	.34	
15	1.07	.53	
15	1.07	.53	
15	1.07	.66	
15	1.07	.73	
15	1.07	.66	
15	1.07	.63	
15	1.07	.63	
15	1.07	.80	.64
16	1.14	.62	
16	1.14	.64	
16	1.14	.60	
16	1.14	.75	.65
17	1.21	.56	.56

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
18	1.29	.81	
18	1.29	.61	
18	1.29	.72	
18	1.29	.64	
18	1.29	.61	
18	1.29	.75	
18	1.29	.94	.72
19	1.36	.73	
19	1.36	.57	
19	1.36	.69	
19	1.36	.98	
19	1.36	1.11	
19	1.36	.73	
19	1.36	.54	
19	1.36	.74	
19	1.36	.83	
19	1.36	.78	.77
20	1.43	.64	
20	1 43	.69	
20	1.43	.72	
20	1.43	.77	
20	1.43	.63	
20	1.43	.63	
20	1.43	.72	
20	1.43	.63	.68
21	1.50	.86	
21	1.50	.71	
21	1.50	.57	
21	1.50	.50	
21	1.50	1.00	
21	1.50	.78	
21	1.50	.64	.72
22	1.57	.55	
22	1.57	.55	
22	1.57	.83	
22	1.57	.57	
22	1.57	.91	
22	1.57	.57	
22	1.57	.97	.70
23	1.64	.82	
23	1.64	.69	
23	1.64	1.21	
23	1.64	.59	
23	1.64	.44	

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
23	1.64	82	
23	1.64	1.08	.80
24	1.71	.75	
24	1.71	.75	
24	1.71	.75	
24	1.71	.75	
24	1.71	1.02	
24	1.71	.72	
24	1.71	.65	
24	1.71	.55	
24	1.71	.89	
24	1.71	1.02	
24	1.71	.75	
24	1.71	.90	.79
25	1.79	1.09	
25	1.79	.72	
25	1.79	1.07	
25	1.79	1.09	
25	1.79	1.50	
25	1.79	.72	
25	1.79	.72	
25	1.79	.93	
25	1.79	.44	.92
26	1.86	.87	
26	1.86	.63	
26	1.86	.87	
26	1.86	.89	
26	1.86	1.28	
26	1.86	.87	
26	1.86	.87	
26	1.86	1.00	.91
27	1.93	.66	
27	1.93	.93	
27	1.93	.66	
27	1.93	.95	
27	1.93	.66	.77
28	2.00	.84	
28	2.00	.58	
28	2.00	.70	
28	2.00	.70	
28	2.00	.70	
28	2.00	1.06	
28	2.00	.70	
29	2.07	.93	

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
29	2.07	.66	
29	2.07	1.08	
29	2.07	.79	
29	2.07	.66	.82
30	2.14	.86	
30	2.14	.62	
30	2.14	.98	
30	2.14	.86	
30	2.14	.62	
30	2.14	.86	.80
31	2.21	.80	
31	2.21	.73	
31	2.21	.84	
31	2.21	1.04	
31	2.21	.95	
31	2.21	.49	.81
32	2.28	1.03	
32	2.28	.59	
32	2.28	.93	
32	2.28	.50	
32	2.28	.75	
32	2.28	1.23	.84
33	2.36	1.16	
33	2.36	.78	
33	2.36	.68	
33	2.36	.78	
33	2.36	.83	
33	2.36	.68	
33	2.36	.78	
33	2.36	.83	
33	2.36	.68	
33	2.36	.87	
33	2.36	.78	
33	2.36	.68	.80
34	2.43	1.04	
34	2.43	.85	
34	2.43	.78	
34	2.43	.90	.89
35	2.50	.93	
35	2.50	.85	
35	2.50	1.13	
35	2.50	.63	
35	2.50	1.00	
35	2.50	1.05	.94

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
36	2.58	.70	
36	2.58	.90	
36	2.58	1.19	
36	2.58	1.29	
36	2.58	.70	.95
37	2.64	1.24	
37	2.64	1.06	
37	2.64	1.16	
37	2.64	.69	
37	2.64	.98	
37	2.64	1.00	
37	2.64	.82	.96
37	2.64	.69	
38	2.71	.76	
38	2.71	1.43	
38	2.71	1.17	
38	2.71	.95	
38	2.71	.79	1.02
39	2.79	.70	
39	2.79	.73	
39	2.79	.84	
39	2.79	.81	
39	2.79	1.26	.87
40	2.86	.60	
40	2.86	.92	
40	2.86	.74	
40	2.86	.78	.76
41	2.93	.82	
41	2.93	.64	
41	2.93	.94	
41	2.93	1.20	
41	2.93	.94	.91
42	3.00	1.20	
42	3.00	.94	
42	3.00	.57	
42	3.00	.66	
42	3.00	.54	
42	3.00	.66	
42	3.00	.81	
42	3.00	1.08	.73
43	3.07	1.14	
43	3.07	.92	
43	3.07	.77	
43	3.07	.80	

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
43	3.07	.52	
43	3.07	.37	.75
44	3.14	.66	
44	3.14	1.16	
44	3.14	1.16	.99
45	3.21	.83	
45	3.21	1.06	
45	3.21	.93	
45	3.21	1.09	.98
46	3.28	.98	
46	3.28	.79	
46	3.28	.89	
46	3.28	.98	
46	3.28	.89	.91
47	3.36	.91	
47	3.36	.74	
47	3.36	.81	
47	3.36	1.01	
47	3.36	.91	
47	3.36	.67	.84
48	3.43	1.20	
48	3.43	.93	
48	3.43	.99	
48	3.43	1.13	
48	3.43	.99	
48	3.43	1.13	
48	3.43	.99	1.05
49	3.50	.63	
49	3.50	.84	
49	3.50	.77	
49	3.50	.84	.77
50	3.57	.71	.71
51	3.64	.76	
51	3.64	.61	
51	3.64	.80	.72
52	3.71	1.04	
52	3.71	.74	.89
54	3.86	1.00	
54	3.86	1.15	
54	3.86	.73	.96
55	3.93	1.06	
55	3.93	.79	
55	3.93	.94	
55	3.93	.94	.94

TABLE XXII (*Continued*)

Total	*Average*	*A. D.*	*C. T. of A. D.'s.*
56	4.00	.56	.56
57	4.07	.77	
57	4.07	.77	.77
58	4.14	.75	
58	4.14	1.41	
58	4.14	1.16	1.11
59	4.21	.93	
59	4.21	1.18	1.06
60	4.28	.47	
60	4.28	.68	.57
61	4.36	1.09	1.09
62	4.43	1.51	
62	4.43	1.02	1.26
64	4.57	.96	
64	4.57	.96	.96
65	4.64	.74	.74
66	4.71	1.04	1.04
68	4.86	.87	.87
70	5.00	.70	.70
71	5.07	.66	.66
72	5.14	.87	.87
73	5.21	.83	.83
76	5.43	1.03	1.03
85	6.07	1.21	1.21
110	7.85	1.02	1.02

TABLE XXIII

Showing the Data for Third and Fourth Grades, Accurates with C. T.'s and Average A. D.'s Combined into Twelve Groups

C. T.	*Average A. D.*
.07 to .43	.40
.50 to .86	.67
.93 to 1.29	.65
1.36 to 1.71	.75
1.79 to 2.14	.13
2.21 to 2.58	.87
2.67 to 3.00	.88
3.07 to 3.43	.92
3.50 to 3.86	.81
3.93 to 4.28	.84
4.36 to 5.21	.90
5.43 to 7.85	1.09

The twelve groups given here were obtained by combining the data of Table XXII so that as nearly as possible the twelve groups would proceed by steps of .36. The data would not submit to this treatment in the last two groups of the twelve.

TABLE XXIV

Attempts, Third and Fourth Grades, Showing Average Performance and Average Variability of Certain Groups of Individuals for the Twenty-eight Minutes. Eighty-five Groups

Number of Columns		*Number of Columns*		*Number of Columns*	
C. T.	*Average of A. D.*	*C. T.*	*Average of A. D.*	*C. T.*	*Average of A. D.*
.28	.55	2.36	.62	4.07	.49
.43	.49	2.43	.52	4.14	.81
.64	.55	2.50	.67	4.21	.60
.78	.34	2.57	.53	4.28	.70
.86	.24	2.64	.41	4.35	.79
1.00	.10	2.71	.67	4.43	.67
1.07	.13	2.78	.54	4.50	.68
1.14	.29	2.86	.59	4.57	.62
1.21	.46	2.93	.41	4.64	.71
1.28	.41	3.00	.42	4.71	.89
1.35	.60	3.07	.68	4.78	1.18
1.43	.56	3.14	.53	4.85	.55
1.50	.58	3.21	.49	4.93	.42
1.57	.57	3.28	.67	5.00	.33
1.64	.47	3.35	.55	5.07	.95
1.71	.58	3.43	.66	5.14	1.08
1.78	.54	3.50	.66	5.21	.42
1.83	.47	3.57	.64	5.28	.60
1.93	.37	3.64	.56	5.36	.62
2.00	.46	3.71	.71	5.43	.65
2.07	.37	3.78	.60	5.50	.64
2.21	.55	3.92	.49	5.64	.53
2.28	.56	4.00	.23	5.78	.58
				5.85	.51
				5.93	.82
				6.00	.57
				6.07	.67
				6.28	.79
				6.43	.72
				6.50	2.14
				6.93	.53
				7.00	.43
				7.43	.92
				8.71	.78
				9.79	6.21
				13.64	3.72

TABLE XXV

Showing Data for the Foregoing Table Combined into Twelve Groups

C. T.	*Average of A. D.*
.28 to 1.71	.46
1.78 to 2.14	.45
2.25 to 2.57	.58
2.64 to 3.00	.51
3.07 to 3.43	.59
3.50 to 3.86	.63
3.92 to 4.28	.55
4.35 to 4.71	.73
4.78 to 5.28	.69
5.36 to 5.70	.67
5.78 to 6.93	.81
7.00 to 13.64	2.41

In this table we proceed as nearly by steps of .36 as possible. The first group and last four groups could not well be made to fulfill this condition. But as this does not affect the mathematical significance of the results, it matters little.

TABLE XXVI

Accuracy, Seventh and Eighth Grades, Showing Average Performance and Average Variability of Certain Groups of Individuals for the Forty-two Minutes of Working. There are 122 Groups

Number of Columns		*Number of Columns*		*Number of Columns*	
C. T.	*Average of A. D.*	*C. T.*	*Average of A. D.*	*C. T.*	*Average of A. D.*
.43	.53	2.38	1.10	3.57	1.05
.48	.59	2.43	1.21	3.62	1.00
.62	.65	2.48	1.07	3.67	.90
.71	.82	2.52	.83	3.71	1.24
.76	.58	2.57	1.30	3.76	1.08
.90	.61	2.62	1.01	3.81	1.15
1.04	.93	2.67	1.04	3.86	1.23
1.29	.92	2.71	.99	3.90	.65
1.33	.51	2.76	.82	3.95	1.11
1.38	.89	2.81	1.05	4.00	1.19
1.43	1.37	2.86	.84	4.05	1.00
1.62	.99	2.90	.90	4.10	1.09
1.81	.77	2.95	.60	4.14	1.09
1.86	1.62	3.00	.76	4.19	.98
1.90	1.03	3.05	1.00	4.24	1.27
1.95	.73	3.09	.97		

TABLE XXVI (*Continued*)

Number of Columns		*Number of Columns*		*Number of Columns*	
C. T.	*Average of A. D.*	*C. T.*	*Average of A. D.*	*C. T.*	*Average of A. D.*
		3.14	1.02	4.29	1.38
2.00	.48	3.19	.71	4.38	1.07
2.05	.78	3.24	.97	4.43	1.12
2.09	.92	3.29	1.04	4.48	.90
2.14	.89	3.33	.83	4.52	1.21
2.19	.95	3.38	1.11	4.57	1.08
2.24	.83	3.43	1.09	4.62	1.15
2.29	1.15	3.48	.89	4.67	1.11
2.33	1.01	3.52	1.21	4.71	1.49
4.76	1.16	5.33	1.02	6.09	1.03
4.81	1.11	5.43	1.81	6.14	1.39
4.86	1.84	5.48	1.26	6.19	.99
4.90	1.61	5.52	1.46	6.28	1.08
4.95	.97	5.57	1.54	6.43	1.30
5.00	.67	5.62	1.10	6.48	1.22
5.05	1.24	5.71	1.01	6.57	.95
5.09	1.14	5.76	1.14	6.62	1.14
5.14	1.14	5.86	.86	6.76	1.32
5.19	1.16	5.90	.87	6.90	1.28
5.24	1.34	5.95	.46	6.95	1.17
5.29	.98	6.00	.86	7.05	.78
				7.14	1.44
				7.19	1.38
				7.48	1.41
				7.57	1.31
				7.67	1.85
				7.71	1.35
				7.80	1.35
				8.00	1.33
				8.14	1.04
				8.52	.98
				8.71	1.59
				9.33	1.49
				11.52	1.65

TABLE XXVII

Showing Data of Foregoing Table Combined into Twelve Groups

C. T.	*Average of A. D.*
.43 to 1.90	.85
1.95 to 2.38	.88
2.43 to 2.86	1.02
2.90 to 3.33	.88
3.38 to 3.81	1.07
3.86 to 4.29	1.10
4.38 to 4.76	1.14
4.81 to 5.24	1.15
5.29 to 5.71	1.28
5.76 to 6.19	.95
6.28 to 6.62	1.14
6.76 to 11.52	1.34

The data of Table XXVI could not be easily handled in a graphic representation, consequently it was combined into twelve groups as in Table XXIII. In this particular table we proceed as nearly by steps of .43 of a column as possible. The first and last groups of the series could not be well made to go by the step.

TABLE XXVIII

Showing Central Tendencies and Corresponding Average A. D. of Columns Attempted by Groups of Individuals of Seventh and Eighth Grades. There are 127 Groups

Number of Columns		*Number of Columns*		*Number of Columns*	
C. T.	*Average of A. D.*	*C. T.*	*Average of A. D.*	*C. T.*	*Average of A. D.*
2.14	.70	5.29	.65	8.00	2.22
2.43	.68	5.33	.62	8.05	1.13
2.76	.58	5.38	.94	8.09	.79
2.81	.61	5.43	.79	8.14	.41
2.90	.61	5.48	1.04	8.24	.67
2.95	1.17	5.52	.77	8.62	1.13
3.00	.29	5.57	.82	8.71	.74
3.05	.54	5.62	.73	8.81	.92
3.09	.53	5.67	.65	8.86	1.12
3.19	1.15	5.71	.92	9.09	.88
3.24	.67	5.76	1.05	9.29	.64
3.29	.62	5.81	.57	9.33	1.27
3.33	.51	5.86	.81	9.38	.93
3.38	.69	5.90	.79	9.52	1.02
3.43	.53	5.93	.66	9.57	.88

TABLE XXVIII (*Continued*)

Number of Columns C. T.	*Average of A. D.*	*Number of Columns* C. T.	*Average of A. D.*	*Number of Columns* C. T.	*Average of A. D.*
3.52	.69	6.00	.72	9.67	.92
3.57	.79	6.05	.83	9.71	.84
3.67	.64	6.09	.64	10.24	1.03
3.71	.90	6.14	.75	10.57	.89
3.76	1.20	6.19	.72	10.62	.91
3.81	.62	6.24	1.14	10.71	.99
3.90	.88	6.29	.91	10.76	.89
3.95	1.14	6.33	1.24	11.19	.81
4.00	.76	6.38	.81	11.43	1.29
4.05	.76	6.43	.83	11.62	2.77
4.09	.67	6.52	.83	11.86	5.14
4.14	.92	6.57	.69	13.09	1.63
4.19	.65	6.62	.95		
4.24	.65	6.67	.70		
4.29	.72	6.71	.72		
4.33	.68	6.76	.65		
4.38	.72	6.86	.61		
4.43	.20	6.90	.63		
4.48	.51	6.95	.71		
4.52	1.10	7.05	.89		
4.57	.59	7.09	.98		
4.62	.69	7.14	.30		
4.67	.74	7.19	.61		
4.71	.57	7.24	.56		
4.76	.55	7.29	.92		
4.81	.99	7.33	.96		
4.86	.68	7.38	.64		
4.90	.45	7.43	.90		
4.95	.78	7.48	1.69		
5.00	.38	7.52	.71		
5.05	.70	7.57	.68		
5.09	.68	7.62	.87		
5.19	.74	7.71	.97		
5.24	1.26	7.76	.88		
		7.82	.83		
		7.95	.88		

The data of Table XXVIII could not be readily handled graphically, and were consequently combined into twelve groups as in Table XXV. In this particular table we proceed as nearly by steps of .7 of a column as possible, but it is difficult to do this because at the beginning and ending of the series—and in the middle —the numbers were so very much scattered. The best grouping was made that could be made. This does not impair the mathematical value of the graph.

TABLE XXIX

Showing Data of Foregoing Table, Combined into Twelve Groups

C. T.	*Average of A. D.*
2.14 to 2.90	.64
2.95 to 4.05	.75
4.09 to 4.76	.66
4.81 to 5.48	.76
5.48 to 6.19	.77
6.24 to 6.90	.82
6.95 to 7.62	.81
7.71 to 8.24	.98
8.62 to 8.81	.93
8.86 to 9.71	.94
10.24 to 10.76	.94
11.19 to 13.09	2.34

The actual facts for the present test are seen on comparing the average variability with the absolute measure of performance. (See the accompanying tables—XXII–XXVIII—which give, for the individuals having the same central tendency, the average of their A. D.'s.) Each of these tables is followed by a regrouping which brings out more fully the tendency of the A. D.'s to increase with the C. T.

In the absence of any universally accepted law, we can take these figures (Tables XXII, XXIV, XXVI, XXVIII) as our basis and see what law of dependence would fit them best. We have assumed that this law would have the form $\frac{Var.}{C.\,T.^{\frac{1}{n}}} = C.$ and find what value of the unknown n will give best agreement with this law. That is, we are to find such a root of the C. T. as will give the most constant quotient when divided into the corresponding variability.

Thus two methods were used for determining what value of n gave the closest approximation to a constant ratio:

1. Find for what value of n the variability of the quotient is least.

2. Observe for what value of n the curve approaches most closely to the horizontal line.

Tables XXX, XXXI, XXXII, XXXIII give the Average, Average Deviation, and Probable Error for each value of $\frac{x}{y^{\frac{1}{n}}}$ in attempts and accurate performance.

TABLE XXX

Showing for Accurates of Third and Fourth Grades the Central Tendency and Average Deviation of Different Values of $\frac{x}{y^{\frac{1}{n}}}$, *where* $x = A.\ D.$, $y = C.\ T.$

Values of $\frac{x}{y}$	Average	.489	A. D.	.302	P. E.	.254
Values of $\frac{x}{y^{\frac{1}{1.9}}}$	Average	.549	A. D.	.1227	P. E.	.104
Values of $\frac{x}{y^{\frac{1}{2}}}$	Average	.559	A. D.	.119	P. E.	.100
Values of $\frac{x}{y^{\frac{1}{2.2}}}$	Average	.569	A. D.	.1065	P. E.	.090
Values of $\frac{x}{y^{\frac{1}{2.3}}}$	Average	.577	A. D.	.1011	P. E.	.085
Values of $\frac{x}{y^{\frac{1}{2.5}}}$	Average	.593	A. D.	.10069	P. E.	.084
Values of $\frac{x}{y^{\frac{1}{3}}}$	Average	.613	A. D.	.088	P. E.	.075
Values of $\frac{x}{y^{\frac{1}{4}}}$	Average	.646	A. D.	.090	P. E.	.076
Values of $\frac{x}{y^{\frac{1}{6}}}$	Average	.691	A. D.	.096	P. E.	.081
Values of $\frac{x}{y^{\frac{1}{8}}}$	Average	.7089	A. D.	.108	P. E.	.091
Values of $\frac{x}{y^{\frac{1}{10}}}$	Average	.7283	A. D.	.111	P. E.	.094
Values of x	Average	.793	A. D.	.150	P. E.	.127

TABLE XXXI

Showing for Attempts of Third and Fourth Grades the Central Tendency and Average Deviation of Different Values of $\frac{x}{y^{\frac{1}{n}}}$

Values of $\frac{x}{y}$	Average	.241	A. D.	.130	P. E.	.012
Values of $\frac{x}{y^{\frac{1}{2}}}$	Average	.376	A. D.	.126	P. E.	.011
Values of $\frac{x}{y^{\frac{1}{3}}}$	Average	.453	A. D.	.130	P. E.	.011
Values of $\frac{x}{y^{\frac{1}{4}}}$	Average	.501	A. D.	.167	P. E.	.016
Values of $\frac{x}{y^{\frac{1}{6}}}$	Average	.560	A. D.	.192	P. E.	.016
Values of $\frac{x}{y^{\frac{1}{8}}}$	Average	.596	A. D.	.212	P. E.	.019
Values of $\frac{x}{y_{10}}$	Average	.615	A. D.	.225	P. E.	.017
Values of x	Average	.700	A. D.	.290	P. E.	.026

TABLE XXXII

Showing for Accurates of Seventh and Eighth Grades the Central Tendency and Average Deviation of Different Values of $\frac{x}{y^{\frac{1}{n}}}$

Values of $\frac{x}{y}$	Average	.329	A. D.	.1431	P. E.	.011
Values of $\frac{x}{y^{\frac{1}{2}}}$	Average	.558	A. D.	.1133	P. E.	.009
Values of $\frac{x}{y^{\frac{1}{3}}}$	Average	.684	A. D.	.1057	P. E.	.008
Values of $\frac{x}{y^{\frac{1}{4}}}$	Average	.766	A. D.	.1196	P. E.	.091
Values of x	Average	1.069	A. D.	.2079	P. E.	.054

TABLE XXXIII

Showing for Attempts of Seventh and Eighth Grades the Central Tendency and Average Deviation of Different Values of $\frac{x}{y^{\frac{1}{n}}}$

Values of $\frac{x}{y}$	Average	.148	A. D.	.048	P. E.	.004
Values of $\frac{x}{y^{\frac{1}{2}}}$	Average	.346	A. D.	.085	P. E.	.006
Values of $\frac{x}{y^{\frac{1}{3}}}$	Average	.467	A. D.	.107	P. E.	.008
Values of $\frac{x}{y^{\frac{1}{4}}}$	Average	.542	A. D.	.116	P. E.	.009
Values of x	Average	.86	A. D.	.304	P. E.	.0228

Explanation

Tables XXXIV, XXXV, XXXVI, and XXXVII give the different values of $\frac{x}{y^{\frac{1}{n}}}$ combined into twelve groups. (See Figures 13, 14, 15, and 16.)

Figures 13 to 16 show graphically the position of each value of $\frac{x}{y^{\frac{1}{n}}}$ on a vertical scale for increasing total performance on a horizontal scale.

TABLE XXXIV

Accuracy, Third and Fourth Grades, Values $\frac{x}{y^{\frac{1}{n}}}$

Total	*Average* x	*Average* $\frac{x}{y}$	*Average* $\frac{x}{y^{\frac{1}{1.9}}}$	*Average* $\frac{x}{y^{\frac{1}{2}}}$	*Average* $\frac{x}{y^{\frac{1}{2.2}}}$	*Average* $\frac{x}{y^{\frac{1}{2.3}}}$	*Average* $\frac{x}{y^{\frac{1}{2.5}}}$
1 to 6	.40	1.67	.82	.79	.74	.69	.72
7 to 12	.67	.99	.82	.85	.80	.81	.78
13 to 18	.65	.59	.62	.62	.62	.63	.61
19 to 24	.75	.48	.60	.60	.62	.63	.64
25 to 30	.83	.42	.59	.59	.61	.62	.64
31 to 36	.87	.36	.55	.56	.59	.60	.62
37 to 42	.88	.31	.51	.52	.55	.56	.60
43 to 48	.92	.28	.49	.51	.53	.55	.57
49 to 54	.81	.21	.34	.42	.45	.46	.49
55 to 60	.84	.20	.40	.41	.44	.46	.48
61 to 73	.90	.19	.40	.41	.44	.46	.48
76 to 110	1.09	.16	.41	.43	.47	.49	.52

TABLE XXXIV (*Continued*)

Total	*Average* $\frac{x}{y^{\frac{1}{3}}}$	*Average* $\frac{x}{y^{\frac{1}{4}}}$	*Average* $\frac{x}{y^{\frac{1}{6}}}$	*Average* $\frac{x}{y^{\frac{1}{8}}}$	*Average* $\frac{x}{y^{\frac{1}{10}}}$
1 to 6	.62	.67	.50	.47	.46
7 to 12	.76	.74	.71	.70	.69
13 to 18	.63	.64	.64	.64	.64
19 to 24	.65	.67	.70	.71	.72
25 to 30	.66	.70	.74	.75	.78
31 to 36	.65	.70	.76	.78	.80
37 to 42	.62	.66	.74	.77	.79
43 to 48	.62	.69	.75	.79	.81
49 to 54	.53	.59	.64	.69	.71
55 to 60	.51	.59	.66	.65	.72
61 to 73	.54	.61	.69	.74	.77
76 to 110	.59	.69	.79	.86	.90

TABLE XXXV

Attempts, Third and Fourth Grades, Values

Total Number of Columns	*Average* x	*Average* $\frac{x}{y}$	*Average* $\frac{x}{y^{\frac{1}{2}}}$	*Average* $\frac{x}{y^{\frac{1}{3}}}$	*Average* $\frac{x}{y^{\frac{1}{4}}}$	*Average* $\frac{x}{y^{\frac{1}{6}}}$	*Average* $\frac{x}{y^{\frac{1}{8}}}$	*Average* $\frac{x}{y^{\frac{1}{10}}}$
4 to 24	.46	.54	.48	.46	.46	.46	.46	.46
25 to 30	.45	.28	.32	.38	.38	.40	.41	.42
31 to 36	.58	.36	.43	.46	.49	.49	.49	.53
37 to 42	.51	.18	.30	.35	.39	.42	.45	.46
43 to 48	.59	.18	.33	.36	.41	.48	.51	.53
49 to 54	.63	.17	.32	.41	.45	.49	.54	.55
55 to 60	.55	.13	.27	.35	.39	.44	.48	.48
61 to 66	.73	.17	.32	.46	.49	.56	.60	.63
67 to 74	.69	.14	.31	.40	.46	.52	.57	.59
75 to 80	.67	.12	.28	.39	.44	.50	.54	.57
81 to 97	.81	.13	.32	.44	.51	.60	.62	.57
98 to 191	2.41	.23	.54	1.11	1.34	1.63	1.78	1.91

TABLE XXXVI

Accuracy, Seventh and Eighth Grades, Values of $\frac{x}{y^{\frac{1}{n}}}$

Total Columns	*Average* x	*Average* $\frac{x}{y}$	*Average* $\frac{x}{y^{\frac{1}{2}}}$	*Average* $\frac{x}{y^{\frac{1}{3}}}$	*Average* $\frac{x}{y^{\frac{1}{4}}}$
9–40	.85	.81	.80	.82	.83
41–50	.88	.41	.60	.67	.75
51–60	1.02	.39	.63	.74	.80
61–70	.88	.21	.52	.60	.64
71–80	1.07	.30	.58	.68	.79
81–90	1.10	.27	.54	.69	.77
91–100	1.14	.25	.53	.69	.78
101–110	1.15	.23	.52	.67	.77
111–120	1.28	.23	.55	.73	.86
121–130	.95	.17	.35	.52	.61
131–140	1.14	.18	.45	.61	.71
141–242	1.34	.17	.48	.67	.80

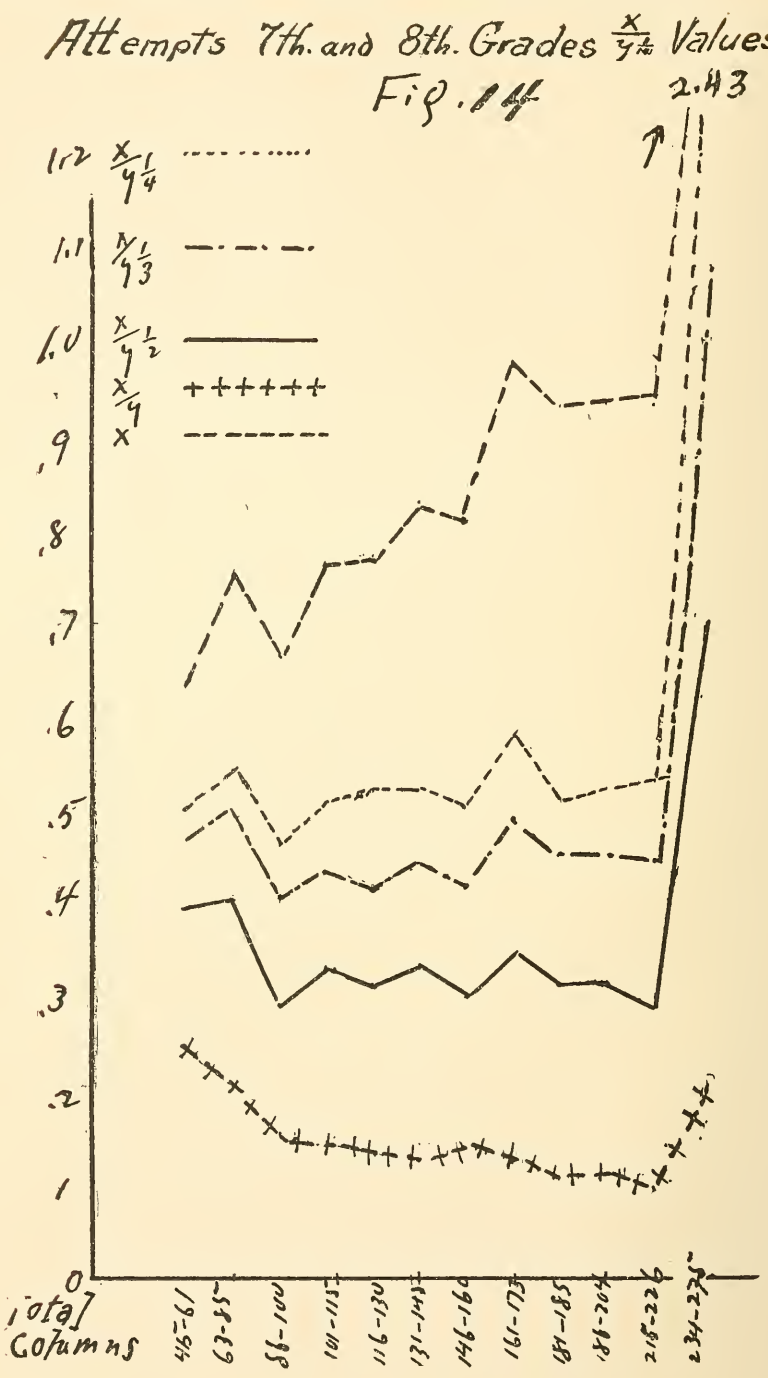
TABLE XXXVII

Attempts, Seventh and Eighth Grades, Values of $\frac{x}{y^{\frac{1}{n}}}$

Total Columns	*Average* x	*Average* $\frac{x}{y}$	*Average* $\frac{x}{y^{\frac{1}{2}}}$	*Average* $\frac{x}{y^{\frac{1}{3}}}$	*Average* $\frac{x}{y^{\frac{1}{4}}}$
45–61	.64	.249	.397	.464	.502
62–85	.75	.215	.403	.495	.542
86–100	.66	.153	.296	.405	.458
101–115	.76	.148	.335	.429	.507
111–130	.77	.131	.316	.424	.524
131–145	.82	.126	.329	.441	.515
146–160	.84	.143	.301	.415	.495
162–173	.98	.122	.346	.488	.581
181–185	.93	.107	.315	.452	.507
186–214	.94	.101	.312	.451	.542
215–221	.94	.089	.289	.437	.527
226–275	2.34	.196	.672	1.011	1.254

Corrected Variability in Attempts

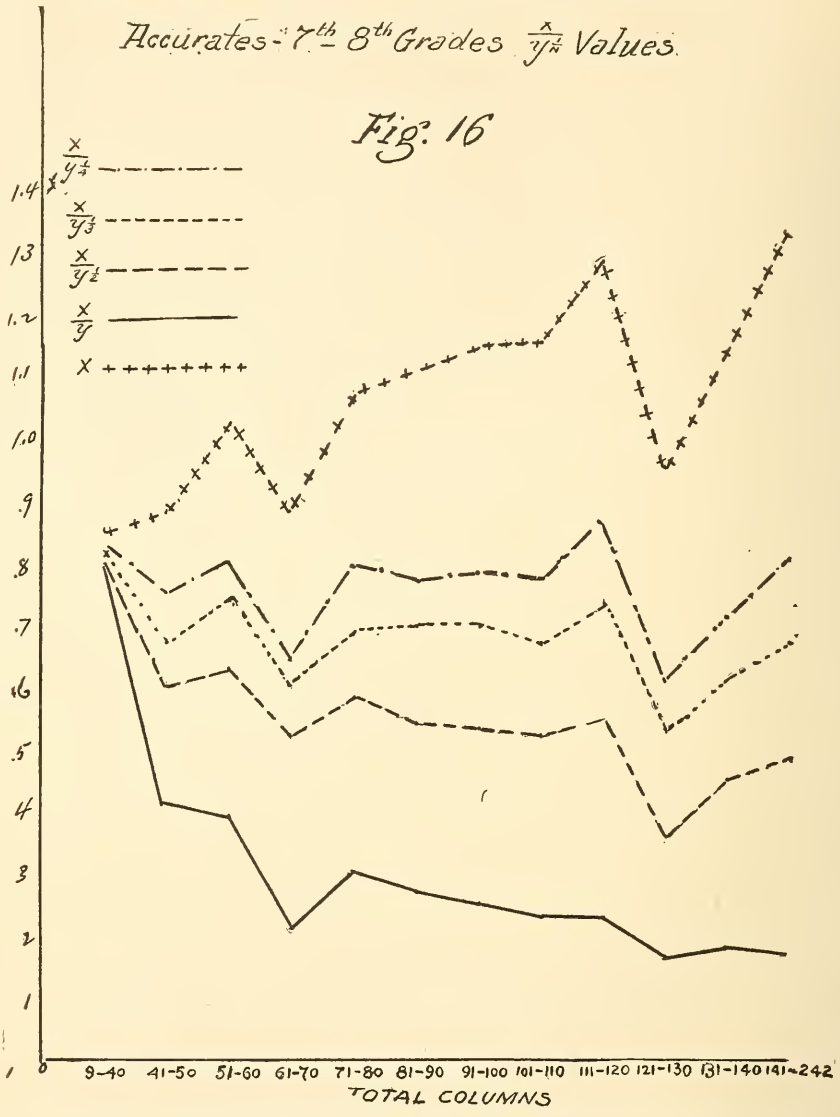
Figures 13 and 14 give the graphical representation for the attempts data of corrected variability for the younger and older groups respectively.


Attempts 7th. and 8th. Grades $\frac{x}{y+\frac{1}{4}}$ Values
Fig. 14
2.43
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
1
0
$\frac{x}{y+\frac{1}{4}}$
$\frac{x}{y+\frac{1}{3}}$
$\frac{x}{y+\frac{1}{2}}$
$\frac{x}{y}$
x
Total Columns
1-9-54
55-63
64-91
92-100
101-113
114-130
131-145
146-160
161-175
176-190
191-205
206-220
221-235

For the Third and Fourth grades attempts, Figure 13 shows the $\frac{x}{y^{\frac{1}{2}}}$, $\frac{x}{y^{\frac{1}{3}}}$ and $\frac{x}{y^{\frac{1}{4}}}$ lines to be nearly horizontal, but the preference goes to the $\frac{x}{y^{\frac{1}{2}}}$ value as it is the most nearly horizontal line of those lines

and besides the $\frac{x}{y^{\frac{1}{2}}}$ series has the smallest average deviation of all the different series of the $\frac{x}{y^{\frac{1}{n}}}$ values.

As to the attempts data of corrected variability for the Seventh and Eighth grades, the $\frac{x}{y}$ values and the $\frac{x}{y^{\frac{1}{2}}}$ values compete most



strongly for the preference. We have chosen here, however, the $\frac{x}{y}$ line, since it appears to be the most nearly horizontal one and the judgment seems justified by the fact that the average deviation of

the $\frac{x}{y}$ series is smaller than that of the other series. But the experimenter thinks that the cause of this inconsistency is due to the fact that some of the subjects at the last attempted too recklessly and thus impaired the data to a small extent. If this had not happened the $y^{\frac{1}{2}}$ value would have been, the writer believes, the chosen value of $y^{\frac{1}{n}}$. (Figure 14.)

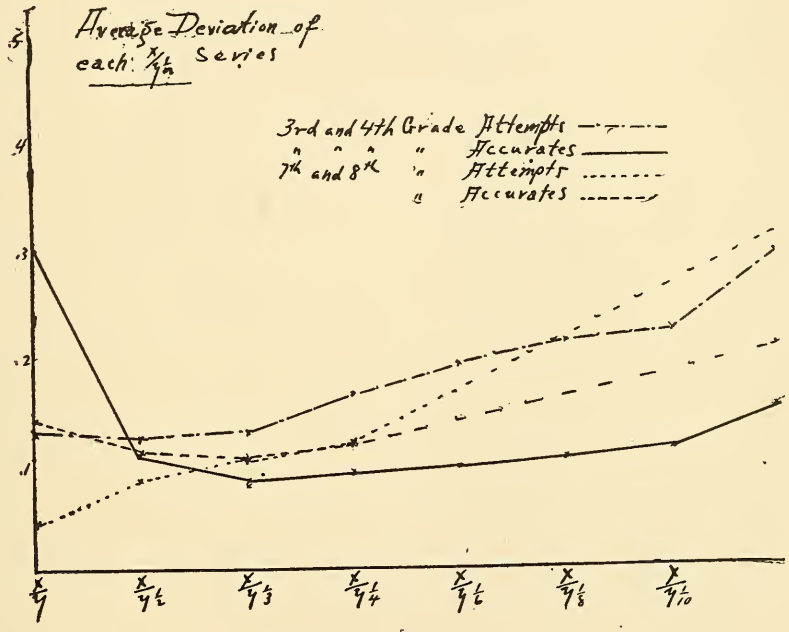


Fig. 17

Figure 17 gives the different positions on a vertical scale for the A. D. of each $\frac{x}{y^{\frac{1}{n}}}$ series for both groups in both attempts and accurates. In the case of the Third and Fourth grades, in attempts, the curve drops lowest for the $\frac{x}{y^{\frac{1}{2}}}$ series; but in accurates it is lowest in the $\frac{x}{y^{\frac{1}{3}}}$ series of values. In the case of the Seventh and Eighth grades, in attempts, the curve is lowest in the $\frac{x}{y}$ series; but in the accurates it goes lowest in $\frac{x}{y^{\frac{1}{4}}}$ as it did in the accurates for the younger children.

Corrected Variability in Accurate Performance

For corrected variability in accurate performance of Third and Fourth grades, Figure 15 shows the lines for values of $\frac{x}{y^{\frac{1}{2}}}$, $\frac{x}{y^{\frac{1}{3}}}$ and $\frac{x}{y^{\frac{1}{4}}}$ (when these are taken respectively as in the expression $\frac{x}{y^{\frac{1}{n}}}$) to be all of them nearly horizontal with the preference for a value between $y^{\frac{1}{3}}$ and $y^{\frac{1}{4}}$ values. We have taken, however, the line representing the $y^{\frac{1}{3}}$ value as the one most nearly horizontal of those in the figure. Also this value has the smallest average deviation from the average. And since these two criteria are those which shall decide in the case, $\frac{Var.}{(C.\ T.)^{\frac{1}{3}}} = C.$ is the formula taken here.

What has been said of the Third and Fourth grades group may also be said of the Seventh and Eighth grades group, Figure 16. While the $\frac{x}{y^{\frac{1}{2}}}$, $\frac{x}{y^{\frac{1}{3}}}$ and $\frac{x}{y^{\frac{1}{4}}}$ lines are all nearly horizontal, the line representing the $y^{\frac{1}{3}}$ values is probably the one most nearly horizontal; and since the $y^{\frac{1}{3}}$ series of values has the smallest average deviation, we give the preference to this value of $\frac{x}{y^{\frac{1}{n}}}$ and so $\frac{Var.}{(C.\ T.)^{\frac{1}{3}}} = C.$ is the formula taken here for this data. However, it is possible here as above that a value lying somewhere between $y^{\frac{1}{3}}$ and $y^{\frac{1}{4}}$ is the correct one for the data of accurate performance.

I think these results agree for the total work and for the accurate work. Total work gives greatest constancy at $\frac{x}{y^{\frac{1}{2}}}$ for Third and Fourth grades and $\frac{x}{y}$ for Seventh and Eighth grades. Accurate work, in the handling of these data gives, or very close to, $\frac{x}{y^{\frac{1}{3}}}$ in both the younger and older groups. It is likely that the data do not lend themselves in their handling to bearing out the theoretical facts. Theoretically this should be $\frac{x}{y^{\frac{1}{2}}}$.

Variability and Performance

Now, if we accept the formula in one case $\frac{Var.}{C.T.^{\frac{1}{2}}} = C.$, or $\frac{Var.}{C.T.^{\frac{1}{3}}} = C.$, as fitting the general run of our facts, having based our correction upon the general run of our own data, of course we cannot find whether those who do less or do more are more variable since what we have done shows them equally variable.

But we can from this graph of the accepted law of dependencies see if there are any eccentricities not according to the normal. We may say that the group has variability in excess of the general run of the data, or that it is close to the general run of the data.

In all the graphs illustrating the accepted law of dependencies we find the last one or two ratios departing, or so tending, from the norm and likewise the first one or two.

In the case of the accurate performances for the younger group, this departure from the norm is only slight (see Figure 15—$y^{\frac{1}{3}}$ line). With the older group the departure at beginning and end is a little more pronounced (Figure 16). But in the case of the attempts data the departure is quite pronounced, especially at the end of the $\frac{x}{y^{\frac{1}{3}}}$ and $\frac{x}{y}$ lines, for both groups. We may then say that this shows a tendency for the subjects who attempt most, and also for those who attempt least, to exceed the normal variability to some extent. This surplus of variability in the slowest and quickest workers is probably due to the fact that they show, more than the medium workers, a progressive change throughout the course of the work, the quickest workers increasing their speed as the work proceeds, and the slowest workers gradually slackening (See pp. 49–54).

VITA

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